NumberSense Companion Workbook Grade 7

Sample Pages (ENGLISH)

Working in the NumberSense Companion Workbook

The NumberSense Companion Workbooks address measurement, spatial reasoning (geometry) and data handling.There are 4 NumberSense Companion Workbooks. With the publication of the NumberSense Companion Workbooks we complete the mathematics curriculum coverage for Grades 4 to 7 (one Companion Workbook per grade). It is our hope that the NumberSense Companion Workbooks will provide children with the same challenges and enjoyment that they get from the NumberSense Workbooks helping them to experience mathematics as a meaningful, sense-making, problem solving activity.

Please note that these sample pages include references from the Companion Workbook Teacher Guide – the actual workbook will not include the Teacher Guide pages. Teachers will be able to download the Teacher Guides, at no charge, from the NumberSense website. You will, however, need to register on the website to access these resources.

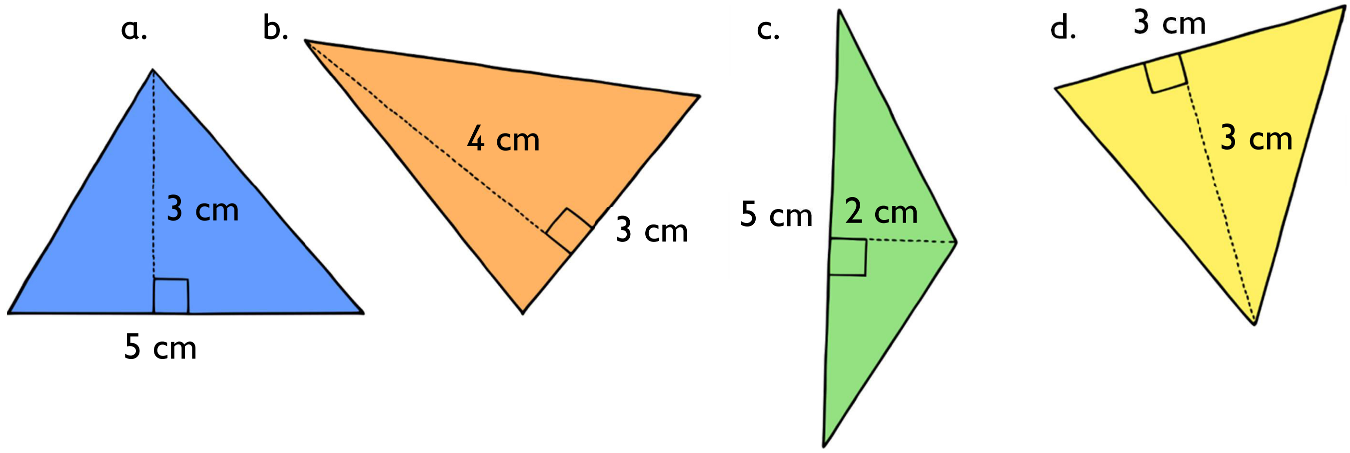
To gain optimal benefit from the workbook series it is critical that children are encouraged to reflect on the tasks that they complete. Teachers (and parents) should ask questions such as:

- Did you notice anything as you completed those activities?
- What helped you to answer the question?
- How is this activity similar to or different from activities that you have already completed?

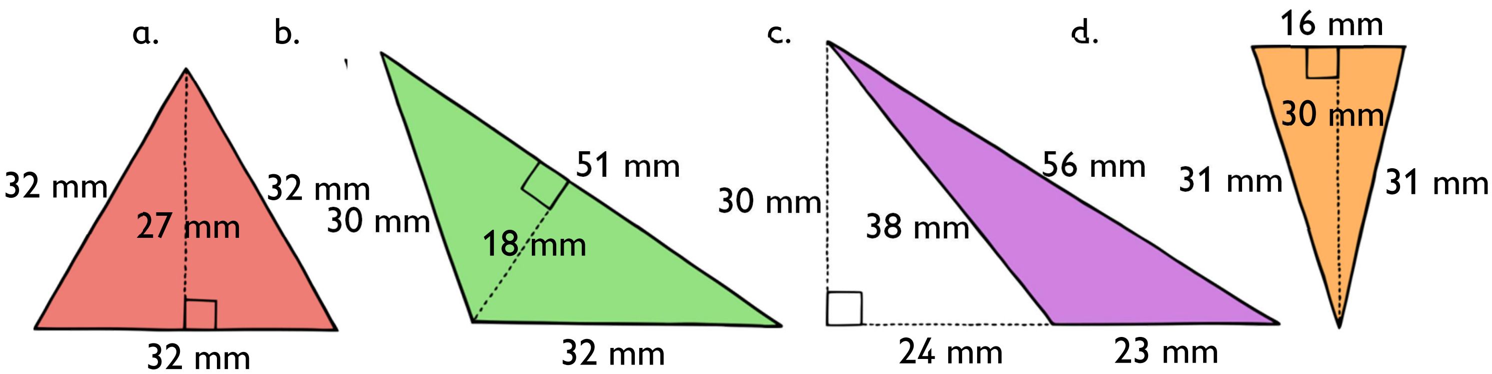
Please mail info@numbersense.co.za for further assistance or information.

The formula for the area of a triangle is: Area = $\frac{1}{2}$ × base × perpendicular height.

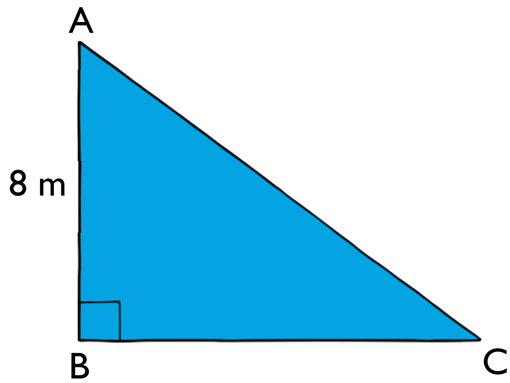
1. Use the formula to determine the area of the triangles.



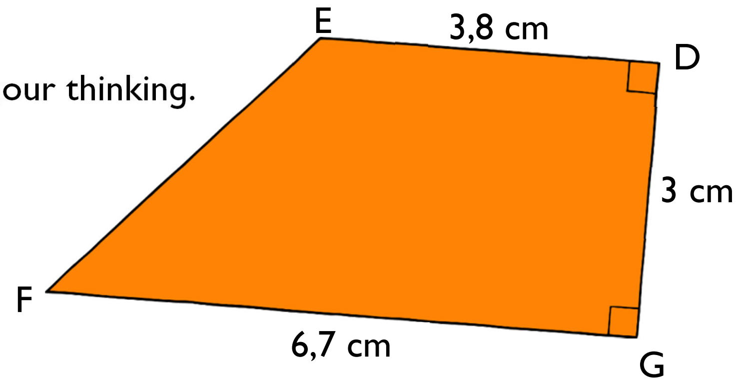
2. Determine the area and perimeter of the triangles.



3. The area of ΔABC is 24 m^2. The perimeter of ΔABC is 24 m. Determine the lengths of BC and AC.



4. Determine the area of DEFG. Explain your thinking.

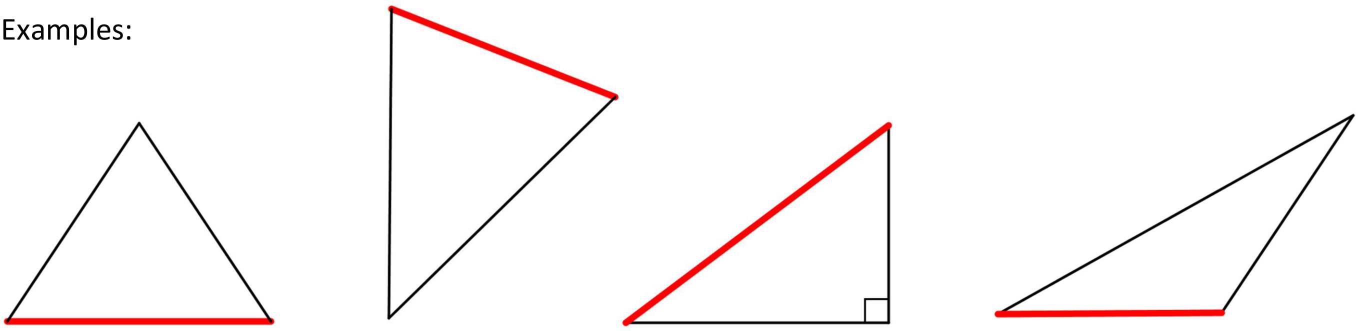


On this page, children solve problems involving area of triangles.

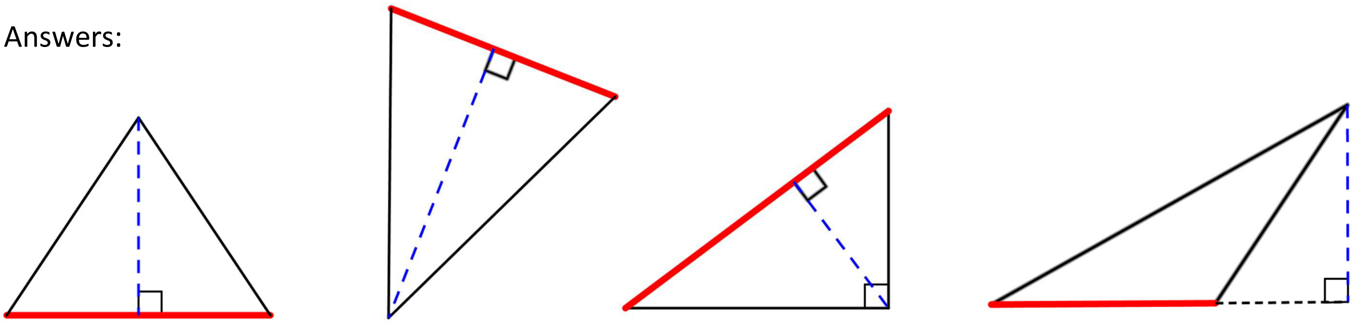
Suggested lesson activities

Children sometimes struggle to see the base and perpendicular height of a triangle. The vocabulary *base* is confusing because we generally tend to think of a base as being the bottom part of an object. Children should get practice early on at seeing bases in various positions and orientations in a variety of triangles. You may want to draw some of these triangles on the board. Ask children to come and draw in the perpendicular height given the red side is a base.

Examples:



Answers:



Throughout this activity, children should discuss their thinking and explain their strategies. For question 4, children should break the polygon up into a rectangle and a triangle.

Answers and discussion

1. a. $7\frac{1}{2}$ cm^2

   b. 6 cm^2

   c. 5 cm^2

   d. $4\frac{1}{2}$ cm^2

2. a. Area 432 mm^2 ; Perimeter 96 mm

   b. Area 459 mm^2 ; Perimeter 113 mm

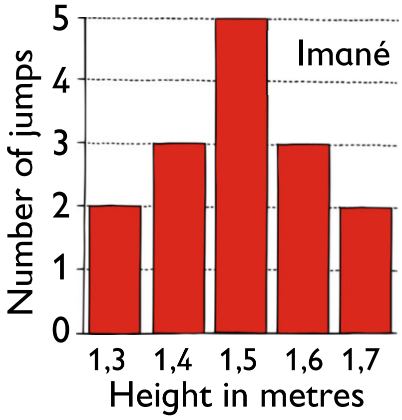
   c. Area 345 mm^2 ; Perimeter 117 cm

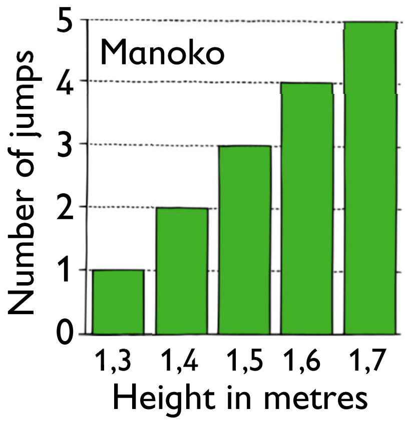
   d. Area 240 mm^2 ; Perimeter 78 mm

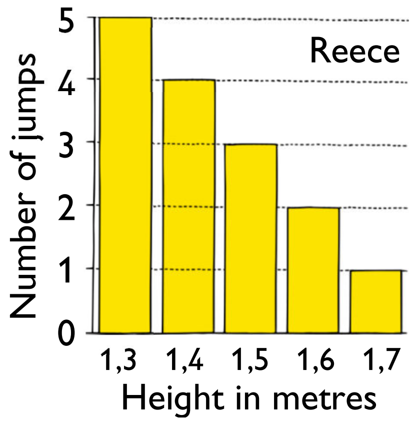
3. BC 6 m and AC = 10 m

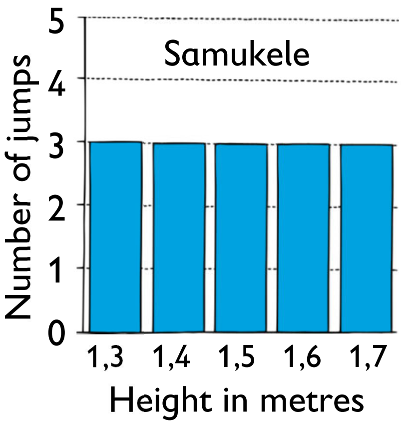
4. Area EDFG 15,75 cm^2

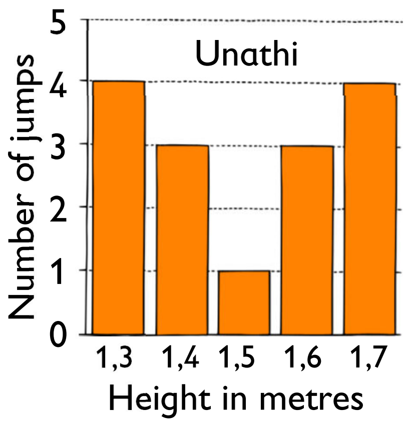
During high jump practice, each athlete made a number of successful jumps at different heights. The graphs show the number of successful jumps at each height for six athletes.

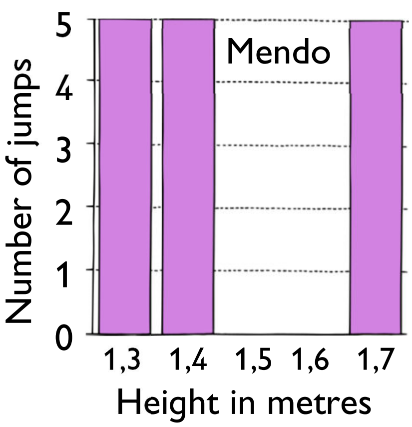












1. How many successful jumps did each athlete make? Did the athletes all make the same number of successful jumps?

2. Which athlete jumped the highest most often?

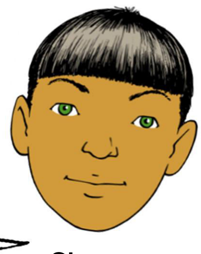
3. How can you use the graph to determine the modal height for each athlete's successful jumps?

4. 

Revat

The median height that every athlete jumped is 1,5 m, because it is always in the middle.

That can't be right. Manoko jumped 15 times. The heights of his jumps in metres were: 1,3; 1,4; 1,4; 1,5; 1,5; 1,5; 1,6; 1,6; 1,6; 1,6; 1,7; 1,7; 1,7; 1,7; 1,7. The middle value is in 8th position which is 1,6 so the median height Manoko jumped is 1,6 m.



Shemy

Who is correct, Revat or Shemy? Explain why the error occurred.

5. Saheel used his calculator to work out the mean height that Imané jumped. This is what he wrote:

$$\text{Mean} = (1{,}3 \times 2 + 1{,}4 \times 3 + 1{,}5 \times 5 + 1{,}6 \times 3 + 1{,}7 \times 2) \div 15$$
$$= 22{,}5 \div 15$$
$$= 1{,}5$$

Is he correct? Discuss.

6. Determine the mode, median and mean (correct to two decimal places) of the successful jumps for each athlete. Complete the table.

	Mode	Median	Mean
Imané			
Manoko			
Reece			
Samukele			
Unathi			
Mendo			

7. For which athletes was the median the same as the mean? What does the graph look like?

8. Can you tell, just by looking at the graphs, whether or not an athlete's mean height will be greater than or less than 1,5 m? Explain.

9. Which athlete do you think will win the school high jump competition? Explain why.

10. Suppose that Samukele completes an additional successful jump that is higher than all his other jumps:
   a. What will happen to the mean of his successful jumps? Discuss.
   b. What happens to the median of his successful jumps? Discuss.

Pages 40 & 41

On these pages, children interpret bar graphs and summarise ungrouped data by determining the means, median and mode as measures of central tendency.

Resources required:

Calculators

Answers and discussion

1. All the athletes made 15 jumps.

2. Manoko and Mendo both jumped the highest (1,7 m) the most (five times).

3. The bars show the number of jumps (or the frequency). The mode is the value that occurs most often so the mode for each athlete is the highest bar.

4. Shemy is correct. Revat did not consider that the athletes jumped these heights a different number of times. He needs to include all 15 jumps when determining the median.

5. Saheel is correct. He has considered every time Imané jumped a particular height and divided this by all the trial jumps (15).

6.

	Mode	Median	Mean
Imané	1,5	1,5	1,50
Manoko	1,7	1,6	1,57
Reece	1,3	1,4	1,43
Samukele	None	1,5	1,50
Unathi	1,3 and 1,7	1,5	1,5
Mendo	None	1,4	1,47

7. The median and mean for Imané, Samukele and Unathi is the same. Their graphs are symmetrical – if you draw a vertical line down the middle, the one side is the mirror reflection of the other side.

8. If there are many values higher than 1,5 m as in Manoko's results, then the mean will be higher than 1,5 m. If there are many values lower than 1,5 m as in Reece's results, then the mean will be less than 1,5 m.

9. Manoko is most likely to win the school high jump competition. He has jumped the highest the most often and the lowest the least often. Both his median height and his mean height are more than the other athletes.

10. a. The mean will be higher because when you include this high value in the calculation it will increase the total by more than 1,5 m.

    b. The median will stay the same. The median will be between position 8 and 9 which is 1,5 m and 1,5 m.

1. Write these numbers in order from smallest to largest.

−70 −7 17 −27

2. Circle the smaller number in each pair.

a. 5 ; −8

b. −12 ; −3

c. 0,5 ; −4

d. $-\frac{1}{5}$; $-\frac{2}{5}$

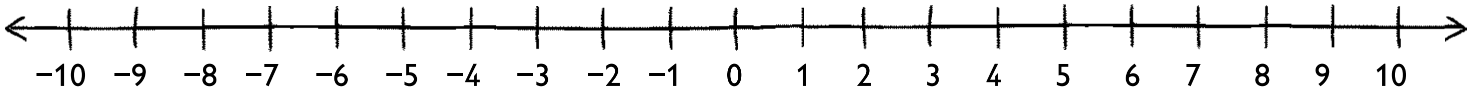
e. $-\frac{1}{6}$; $-\frac{1}{3}$

f. $-\frac{3}{4}$; $-\frac{8}{9}$

3. Place these numbers on the number line as carefully as you can.

0 $-\frac{1}{2}$ 2,25 3 −1

4. Study the number line.



What number is:

a. 2 more than −6

b. 2 less than −6

c. 5 more than −3

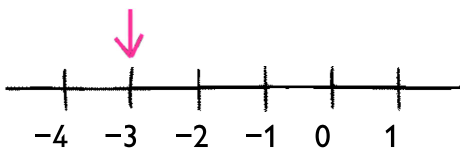
d. 5 more than −9

e. 5 less than −3

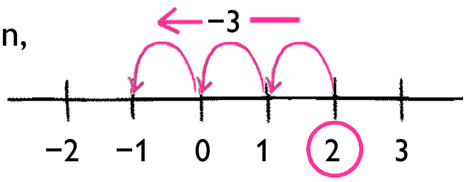
f. 5 more than −5

The symbol '–' has two meanings in mathematics:

When used with a number it is used to indicate that we mean the negative number, for example −3 means negative 3 which is placed on the number line as indicated.



When used as part of an expression it indicates an operation, for example 2 – 3 means subtract 3 from 2 or 2 minus 3.



We write:	We say:
2 – 3	"two minus three"
−2 + 5	"negative two plus 5"
−2 – 8	"negative two minus eight"
−5 – (−3)	"negative five minus negative 3"

5. Calculate the value of each expression. Explain how you can use a number line to help you.

a. 5 – 8

b. −5 + 8

c. −1 – 4

d. −1 + 4

Page 62

On this page, children count forwards and backwards in integers and compare integers.

Suggested lesson activities

In this activity, children order integers and are introduced to adding and subtracting with positive integers even though the solution may be negative. Children should be aware that the "minus" sign now has two different meanings as opposed to the single meaning it has held for them so far. Teachers should make sure that when children are talking about what they are doing, they use the vocabulary correctly, for example, $-8 - 5$ is negative 8 minus 5, not minus 8 minus 5 or negative 8 negative 5.

We encourage children to use the number line to solve these problems using a "count back" strategy in the same way that they have used it since Grade 1. In the calculation $5 - 8$, they start at 5 and count back 8 units. It helps if children understand addition as "counting on" and subtraction as "counting back". Avoid teaching any methods/rules that detract from this meaning. Encourage children to discuss anything they observe.

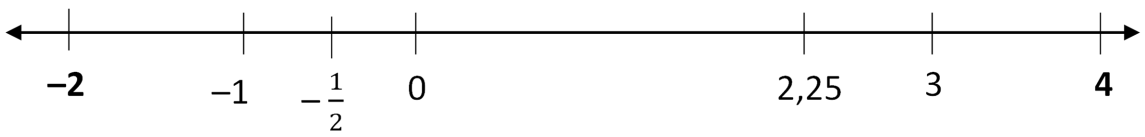
Answers and discussion

1. $-70;\ -27;\ -7;\ 17$

2. a. -8 c. -4 e. $-\frac{1}{3}$

   b. -12 d. $-\frac{2}{5}$ f. $-\frac{8}{9}$

3.



4. a. -4 d. -4

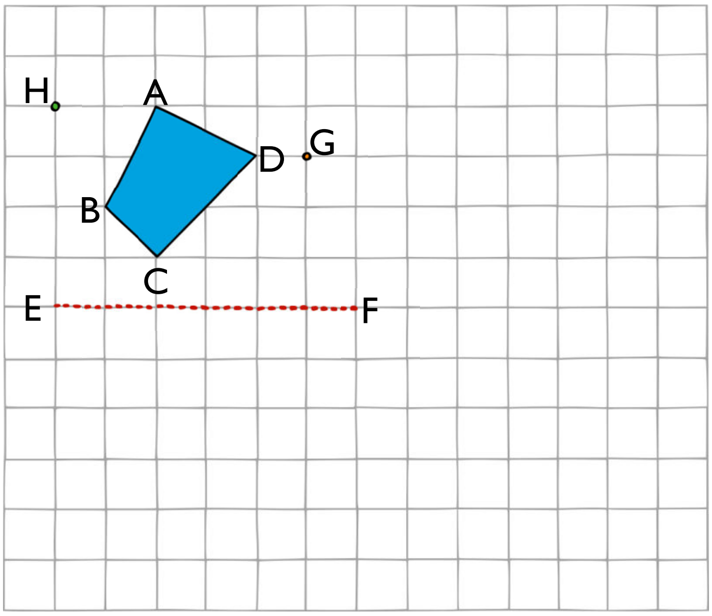
   b. -8 e. -8

   c. 2 f. 0

5. a. -3 c. -5

   b. 3 d. 3

1. a. Translate ABCD 5 units right and 4 units down. Label the image A'B'C'D'.
   b. Reflect ABCD about EF. Label the image A"B"C"D".
   c. Rotate ABCD 180° about G. Label the image A'''B'''C'''D'''.
   d. Enlarge ABCD from H by a scale factor of 3. Label the image A''''B''''C''''D''''.



   e. What do you notice about the different images of ABCD? Discuss.

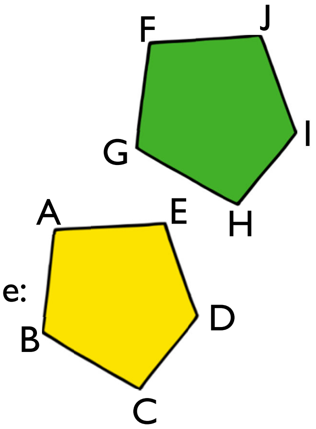
Two figures are said to be congruent if they are the same in both shape and size.

Two figures are said to be similar if both of the following conditions are true:

- The corresponding angles of the two shapes are equal.
- The lengths of the corresponding sides of the two shapes are in the same proportion.

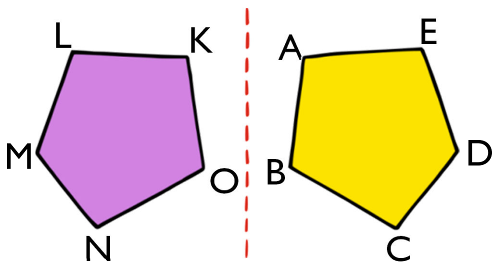
2. a. Which of the images in question 1 are congruent to ABCD? Explain.
   b. Which of the images in question 1 are similar to ABCD? Explain.

3. FGHIJ is the image of ABCDE from a translation.



   a. For each side, name the corresponding/equal side in the image:
      - AE
      - AB
      - CD
   b. For each angle, name the corresponding/equal angle in the image:
      - $B\hat{A}E$
      - $B\hat{C}D$
      - $E\hat{D}C$

4. KLMNO is the image of AEDBC from a reflection.



   a. For each side, name the corresponding/equal side in the image:
      - AE
      - AB
      - CD
   b. For each angle, name the corresponding/equal angle in the image:
      - $B\hat{A}E$
      - $B\hat{C}D$
      - $E\hat{D}C$

On this page, children recognise and describe the properties of similar and congruent figures and the differences between them.

Suggested lesson activities

Children have done a few activities where they have transformed shapes and have been encouraged to reflect on the size of the corresponding angles and length of corresponding sides. Question 1 serves as a consolidation and an opportunity to discuss what children should have noticed from previous exercises, i.e.

When shapes are translated, reflected and rotated:

- The angles of the image are equal to the corresponding angles of the shape and
- The sides of the image are the same length as the corresponding sides of the shape.

Shapes that are translated, reflected and rotated are congruent. Rotations, translations and reflections are known as rigid transformations because the properties of the shape being transformed are unchanged.

When shapes are enlarged (or reduced):

- The angles of the image are equal to the corresponding angles of the shape and
- The ratio of the sides of the image to the corresponding sides of the shape are in the same ratio (sides are in proportion).

Shapes that are enlarged (or reduced) are similar. Enlargements and reductions are known as semi-rigid transformations because some of the properties of the shape being transformed are changed.

Note according to the definitions in the text, all congruent figures are also similar (with the ratio between the corresponding sides equal to 1). This is interesting but not important to raise unless some of the children make this observation. It is a little like the relationship between squares and rectangles: all squares are rectangles, but not all rectangles are squares – all congruent figures are similar, but not all similar figures are congruent.

Encourage children to look for and become aware of what is meant by the corresponding angles and sides in a figure and its image. A side in a figure and the matching side in the image are called corresponding sides. For example, $ABCDE$ (in question 4) is reflected about the vertical line shown. Side LM is the image of side ED – we say that LM and ED are corresponding sides in the figure and its image. Similarly, $K\hat{O}N$ is the image of $A\hat{B}C$ and we say that $K\hat{O}N$ and $A\hat{B}C$ are corresponding angles in the figure and its image. Notice also how side ED is opposite $A\hat{B}C$ in the figure and LM is opposite $K\hat{O}N$ in the image – not only do sides and angles in the figure and its image correspond, but the relationships between the different sides and angles in a figure are preserved in the image.

When an image is named, the order of the letters is important. $KLMNO$ is the image of $AEDCB$. The first vertex K corresponds to A, the second vertex L corresponds to E etc. This is a convention and it is important that as teachers we are careful with this from the start -- children will come to realise this with time.

In questions 3 and 4 children should start to notice that when a shape is translated it is easy to determine which sides or angles are corresponding because the orientation of the shape does not change. However, when a shape is reflected (or rotated), it is not as easy to determine which sides and angles are corresponding.

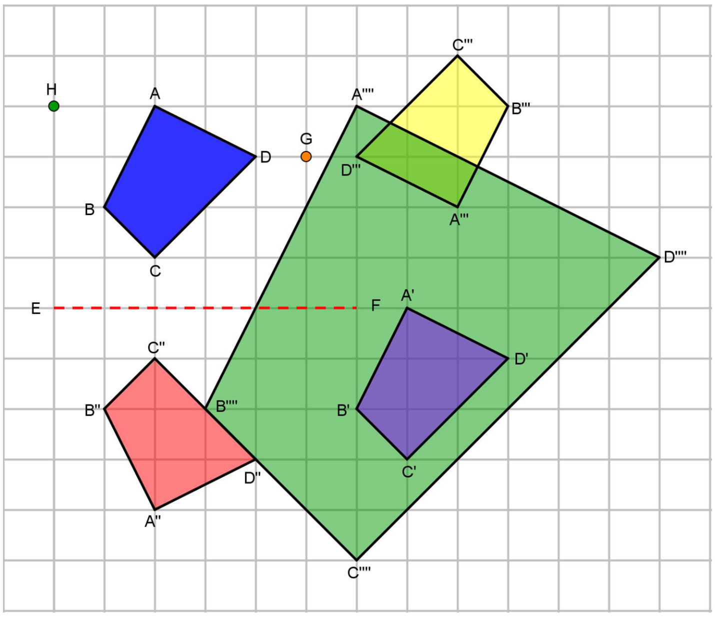
Answers and discussion

1. a.

b.

c.

d.



e. Images $A'B'C'D'$, $A''B''C''D''$ and $A'''B'''C'''D'''$ are the same as $ABCD$. The size and shape has not changed. The corresponding angles are the same size and the corresponding sides are the same length.

$A'B'C'D'$ also has the same orientation as $ABCD$, i.e. $\hat{A}$ and $\widehat{A'}$ is on top.

Image $A''''B''''C''''D''''$ is the same shape as $ABCD$, but it is larger. The ratio of the each side of $ABCD$ to the corresponding side of $A''''B''''C''''D''''$ is $1 : 3$, i.e. the sides are in proportion.

2. a. $A'B'C'D'$, $A''B''C''D''$ and $A'''B'''C'''D'''$ because their corresponding angles and corresponding sides are equal to those of $ABCD$.

b. $A''''B''''C''''D''''$ because their corresponding sides are in proportion.

3. a. $AE = FJ$, $AB = FG$ and $CD = HI$

b. $B\hat{A}E = G\hat{F}J$, $B\hat{C}D = G\widehat{H}I$ and $E\widehat{D}C = J\hat{I}H$

4. a. $AE = KL$, $AB = KO$ and $CD = NM$

b. $B\hat{A}E = O\widehat{K}L$, $B\hat{C}D = O\widehat{N}M$ and $E\widehat{D}C = L\widehat{M}N$

1. Complete the table and create a flow diagram and rule for each line. Use $\square$ to represent the input number in your rule. Some of the answers are already provided.

Input no.	1	2	3	4	Flow diagram	Rule
Output no. 1	3	6	9		Input → $\boxed{\times 3}$ → Output	
Output no. 2	5	8				Output no. = $\square \times 3 + 2$
Output no. 3	9				Input → $\boxed{+2}$ → $\boxed{\times 3}$ → Output	

Algebra helps us to write rules or formula in a more compact or efficient way. In algebra we use variables in the place of the unknowns: $\square$ and the words input and output etc.

For example, Output no. = $\square \times 3 + 2$ can be written, using algebra, as $o = 3i + 2$ or as $y = 3x + 2$.

In algebra, as in all writing, there are conventions:

- Instead of words, letters are used to represent the variables. In this example i and x represent the input number and o and y represent the output number
- $3a$ means $3 \times a$ or $a \times 3$.

2. Match each flow diagram with the corresponding algebraic formula.

a. $a \rightarrow \boxed{\times 4} \rightarrow \boxed{+ 3} \rightarrow b$

c. $a \rightarrow \boxed{\times 3} \rightarrow \boxed{+ 4} \rightarrow b$

b. $a \rightarrow \boxed{+ 4} \rightarrow \boxed{\times 3} \rightarrow b$

d. $a \rightarrow \boxed{+ 3} \rightarrow \boxed{\times 4} \rightarrow b$

A. $b = 3a + 4$ B. $b = 3(a + 4)$ C. $b = 4a + 3$ D. $b = 4(a + 3)$

3. Write an algebraic formula for each flow diagram.

a. $x \rightarrow \boxed{- 10} \rightarrow \boxed{\times 5} \rightarrow y$

d. $x \rightarrow \boxed{\div 2} \rightarrow \boxed{+ 5} \rightarrow y$

b. $n \rightarrow \boxed{\times 6} \rightarrow \boxed{- 8} \rightarrow m$

e. $x \rightarrow \boxed{+ 5} \rightarrow \boxed{\div 2} \rightarrow y$

c. $j \rightarrow \boxed{+ 7} \rightarrow \boxed{\times 10} \rightarrow k$

f. $x \rightarrow \boxed{- 10} \rightarrow \boxed{\div 4} \rightarrow y$

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On this page, children determine, analyse and interpret equivalence of different descriptions of the same relationship or rule presented in flow diagrams, tables and algebraic formulae.

Suggested lesson activities

Children working in NumberSense Workbook 24 have had opportunity to write rules for flow diagrams using placeholders to represent the input number. In this lesson, children are introduced to using letters to stand in the place of variables, i.e. input and output numbers. Children can use any letter to represent the input or output number. Teachers may want to highlight that as we move from expressions, rules and formulae with words and boxes to algebraic representations we lose some of the detail.

Encourage children to discuss the order of operations as they complete this exercise. If addition or subtraction are done first in the flow diagram, then this should be shown using brackets in the formula.

Answers and discussion

1.

Input no.	1	2	3	4	Flow diagram	Rule
Output no. 1	3	6	9	12	Input → $\boxed{\times 3}$ → Output	Output no. = $\square \times 3$
Output no. 2	5	8	11	14	Input → $\boxed{\times 3}$ → $\boxed{+2}$ Output	Output no. = $\square \times 3 + 2$
Output no. 3	9	12	15	18	Input → $\boxed{+2}$ → $\boxed{\times 3}$ → Output	Output no. = $(\square + 2) \times 3$

2. a. C

   b. B

   c. A

   d. D

3. a. $y = 5(x - 10)$

   b. $m = 6n - 8$

   c. $h = 6(g - 7)$

   d. $k = 10(j + 7)$

   e. $y = \frac{x}{2} + 5$

   f. $y = \frac{x + 5}{2}$

   g. $y = \frac{x - 10}{4}$

   h. $y = \frac{x}{4} - 10$