Brombacher
& Associates

Developing the Fraction Concept

A workshop for Foundation Phase teachers

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Contents

Three crucial factors

For the effective development of number sense and children's computational methods we need to pay attention to three crucial factors:

- Developing children's **understanding of number** through counting and manipulating numbers in various ways, measuring, number games, etc.;
- Using **meaningful problems** to introduce the basic operations and concepts and to support the development of increasingly sophisticated computational methods;
- Encouraging **discussion** by children to explain and compare their methods. It is only through discussion that children think about what they have done and try to understand what others have done. Through discussion they come to see the patterns, properties and relationships that underpin numeracy/mathematics and mathematical thinking.

Children must continuously be challenged to think, to reason and to make plans.

Different kinds of knowledge

The different types of knowledge (distinguished by Piaget) shed light on the processes by which the child learns about number – physical knowledge, social knowledge and logico-mathematical (conceptual) knowledge.

Physical knowledge

This is the kind of knowledge which children acquire through interaction with the physical world – e.g. through observing and handling objects

Physical knowledge is derived from concrete experiences – touching; using; playing with; and acting on concrete/physical material. Children need a lot of concrete experiences in the numeracy/mathematics classroom to develop their physical knowledge of number by counting concrete apparatus. It is through counting physical objects that children develop a sense of the size of numbers: 50 takes longer and more actions to count than 5 does but 250 takes a lot more. Five counters can be held in one hand; 50 in two hands; while 250 require a container – there are too many for our hands. Five counters look different from two counters.

Counting physical objects like counters is called rational counting – the counters are physically handled and moved from one place to another. The children observe the pile of counter grow as they count them.

Drawings are are also objects in the sense that the counters described above are. Drawings are representations of the physical world.

The implication of physical knowledge for the Foundation Phase numeracy classroom is very simply that there must be both concrete apparatus (counters, shapes such as building blocks and other construction materials; and measuring apparatus) and the opportunity for children to work/play with the apparatus. It is the teacher's responsibility to provide the materials and the time for children to use them.

Social knowledge

The number five is an unproblematic concept for an adult who has known and used the word for many years; they can imagine five items and can even calculate with five without having to recreate the number using physical counters or representations in their minds. However, if we take a moment to reflect and think about this then we realise that the word five has no intrinsic properties that hint at the number of items it represents. So it is with people's names, place names, days of the week and months

of the year. The words we use to describe these are all “names” that we have assigned – and because the people in our community (society) all associate the same thing with the same name (word) we are able to communicate with each other. In order to know these names (this social knowledge) we need firstly to be told them and secondly to remember them.

We refer to knowledge that must be both told and remembered as social knowledge. The only way in which we can acquire this kind of knowledge is to be told it.

The implication of social knowledge for the classroom in general and the Foundation Phase classroom in particular is that teachers have to tell (teach) this to knowledge to children. They have to introduce children to the vocabulary.

The way in which we write the number symbols, indeed writing in general, is a socially agreed on conventions/habit and once more it is the job of the teacher in the Foundation Phase, in particular, to introduce children to these conventions.

In terms of numeracy/mathematics the words “addition, subtraction, division and multiplications” and the symbols that we use to denote them are all examples of social knowledge. Children “add” and “subtract” quite naturally without knowing the labels for the action that they are performing. When a mother ask her child to help her in the kitchen and says “I need 8 potatoes altogether, I already have three here, please get me the rest” then the child will respond to the question in one of two ways, either he/she will count on: “four, five, six, seven, eight” touching the potatoes while counting or he/she will count out eight potatoes from the packet and put three back. The first action could be called addition while the second subtraction. The point is that the child performs the actions without any knowledge of the word that the observer has to describe it.

Of concern, however, is the practice of many teachers of teaching as social knowledge the kinds of things that children can come to find out by themselves.

Conceptual (logico-mathematical) knowledge

When children reflect on activities (with or without physical objects) and begin to see patterns, relationships, regularities and irregularities within and between the numbers and the operations; they are constructing what is known as logico-mathematical knowledge. Logico-mathematical knowledge is internal knowledge and is constructed by each individual for themselves.

The teacher’s role in the development of children’s logico-mathematical knowledge is two-fold. On the one hand the teacher is responsible for creating activities and situations (problems) that will reveal the underlying structures of numbers, operations, and relationships. On the other hand the teacher needs to actively encourage children to reflect on what they are doing and what they are thinking – helping them to express these ideas in words so that they can explain their actions to others; discuss their respective methods; and even argue about the validity of each.

Since the teacher is unable to teach logico-mathematical knowledge through direct instruction (although it is tempting to try to do so – see the discussion of social knowledge above), one of the most important tasks of teaching is to design situations from which children can construct/develop their logico-mathematical knowledge. That is, the teacher needs to ask the question when designing a lesson/task: “What do I want children to learn from this situation/problem/activity?” Having established what it is that the teacher wants children to learn they then need to shape the situation/problem/activity in a way that will provoke children to “see” the patterns and structures. Then, both during and on completion of the activity, the teacher needs to facilitate reflection on the activity by the child – it is this reflection more than anything else that will provoke the development of logico-mathematical knowledge.

Problem solving

In this section we deal with problem solving as well as the critical role of discussion.

In addressing this topic we will deal with the following questions:

- What is the role of problem solving in the learning of mathematics?
- What is meant by suitable problems? What types of problems should be posed?
- What is the role of discussion?

Children’s mathematical experiences should have a starting point in their own world.

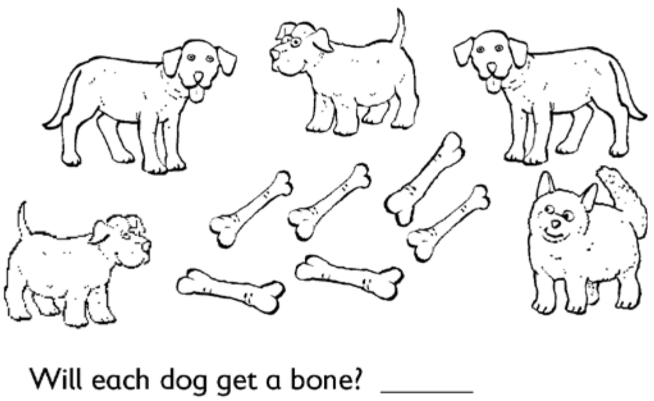
What is the role of problem solving in the learning of mathematics?

Problems are often thought of as a reason for studying mathematics. Mathematicians solve problems. In order to solve the problems that mathematicians solve they need to be able “to do” mathematics. That much is obvious – mathematics is the tool of the mathematician.

What is not always as well appreciated is the idea that problems also provide a way to introduce children to mathematics. Learning mathematics by solving problems! As much as this is true for mathematics at all levels, it is particularly true for children in the early years.

Consider for a moment a young child, say 4 years old. The child’s mother gives her some sweets with the instruction to share these with her little brother. Chances are good that she will be able to make a plan that will result in the fair/equal sharing of the sweets with her brother. Her strategy may well involve a “one for you, one for me, one for you, one for me, ...” dealing out of the sweets until the sweets are exhausted. In mathematical terms the girl has divided the sweets by two and the number of sweets that the girl and her brother each get can be determined by calculating the (number of sweets) ÷ 2. In mathematical terms the girl has completed a division problem and yet she may well be unable to count and may never have heard the word division!

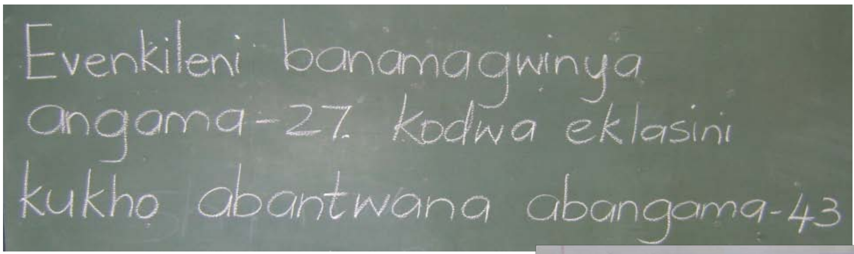
Young children have a natural ability to solve problems and this problem solving ability can, and should, be used to good effect in the introduction of mathematics.



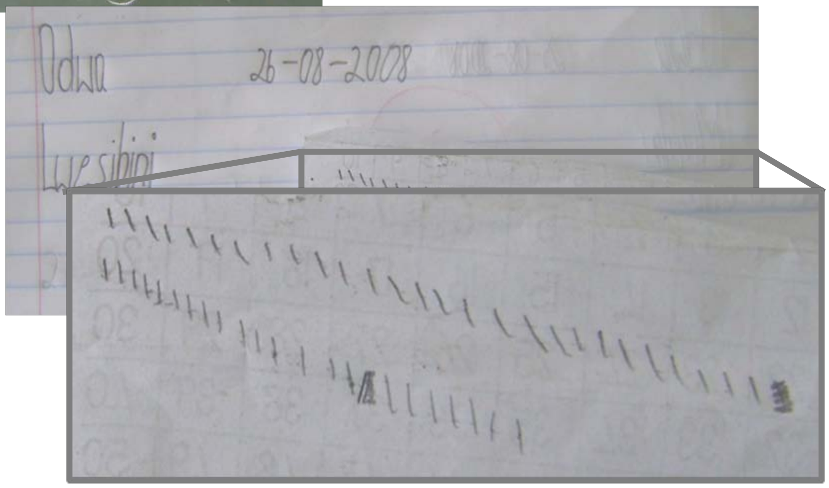
(Grade 1 Workbook 1 page 1)

Not only does taking advantage of children’s natural problem solving ability provide a way of introducing mathematics and in particular the four basic operations (addition, subtraction, multiplication and division) of the Foundation Phase, but it also helps children to see value in the mathematics they are doing. Seeing value in what they are doing, in turn, gives children confidence and this confidence is important in helping children make a success of mathematics!

Consider another problem – the problem involving the dogs and the bones alongside. Children often solve this problem by simply drawing lines connecting the dogs and the bones, and in so doing concluding that there are enough bones for each dog to get their own bone. Children can solve this problem without knowing number names, number symbols, the words addition and subtraction and, or for that matter, before they know the symbols representing these operations.



One more problem: Consider a problem used with children in a Foundation Phase class. The problem states: The tuckshop has made 27 amagwinya (vetkoek). There are 43 children in the class. Are there enough amagwinya for each child to get one? After an enthusiastic classroom discussion it was agreed that there were not enough amagwinya and the children were asked to determine how many more were needed.



Odwa solved the problem by first drawing 27 stripes to represent the 27 children in the class and then he drew a large number of extra stripes. He then counted on from 27: 28, 29, 30 … 43 and highlighted the 43rd stripe. Finally he counted how many extra amagwinya (stripes) were needed and concluded that 16 more amagwinya needed to be made. Mathematically, we can summarise Odwa's solution as follows: 27 + 16 = 43. That is, we can think of Odwa as having added on from 27 to 43 and in so doing to have established that 16 additional amagwinya were needed.

Asavelo solved the problem by first counting out 43 counters. Next she counted out 27 from the 43 – as if she was giving amagwinya to those whom she could give to. Finally she counted the remaining counters and established that she still needed 16 amagwinya for the remaining children. Mathematically, we can summarise Asavelo's solution as follows: 43 – 27 = 16. That is, we can think of Asavelo as having subtracted 27 from 43 to establish that 16 additional amagwinya were needed.

The important point that this illustration makes is that the problem was not so much an addition problem or a subtraction problem, but rather a problem that could, in mathematical terms, be solved by means of both addition and subtraction. After a number of similar problems and appropriate discussion of both Odwa's and Asavelo's approaches the teacher's role becomes one of introducing the mathematical vocabulary and notation associated with these perfectly natural problem solving strategies.

Teachers can choose to either introduce a lesson (in the Foundation Phase) on addition and the rules and methods associated with the procedure, or to present children with problems – problems that will

provoke the children to perform an action that is referred to as addition and then it remains for the teacher to introduce the vocabulary, symbols and conventions associated with the perfectly natural procedure.

There is one more observation to be made. Notice that both Odwa and Asavelo were confident about their answers. They could check them from the context. They had accounted for 43 children, they had used up the 27 amagwinya available and they could justify the need for an additional 16 amagwinya. For many children the only way they have of knowing whether or not their answer to the question: $43 - 27 = \boxed{}$ or $27 + \boxed{} = 43$ is correct, is to ask their teacher!

It goes without saying that as Odwa and Asavelo's number sense improves so they will rely less on stripes and counters and be able to use numbers with greater confidence in solving the same problems.

It is our strongly held conviction that children who learn mathematics through problems will see the value of what they are doing and be able to make sense of what they are doing far more so than those who don't.

Problems not only introduce children to the mathematical operations but problems also play an essential part in the development of computational methods

The structure of the problem initiates the first primitive method that the child constructs to solve that particular problem. As children solve problems over a long period of time, their number concept develops and their methods are refined. With time then develop a more integrated understanding of the operations and the strategy they use will become more sophisticated.

What is the role of discussion?

Interaction about what children have done is important while learning mathematics, because it stimulates reflection and enables them to function at a higher level.

In order to optimize the role of discussion in teaching and learning, a specific classroom culture should be established. This could be seen as a didactical contract, where teachers and learners communicate and negotiate their expectations and obligations.

- The learners must know that they may use any method to solve a problem, but they are expected to explain their ideas and methods to the teacher and to the others in the group or class. They are also expected to listen to the explanations of others in their group or class. Only an explanation that is clear and understood should be accepted. The way in which the teacher handles (and accepts or does not accept) an explanation, indicates that a good, acceptable explanation is one that is understood by everybody in the group or class.
- Learners soon learn to listen critically to one another's explanations and challenge it when it is not understood or clear. This type of interaction (argumentative) is the most powerful for learning to take place. The teacher facilitates guides and gives structure to the interaction and makes the ideas that come up in the discussion explicit for all the learners in the class.
- Because children are expected to make sense of mathematics by working on and solving problems, it is essential that the mathematical ideas which they construct during the process are put "in public". This makes it possible for the teacher to detect any possible errors in thinking or misconceptions that may exist in the child's mind. It also gives the teacher the opportunity to check whether the child uses the conventions of mathematics and terminology correctly when communicating about mathematics and recording their methods.

Fractions – general remarks[1]

The **common fraction** notation for fractions is used less frequently in everyday life than it was before; as a result children are not as used to it as they are to whole numbers. When common fractions are used, it is mostly in an inexact way - young children may therefore call any big piece of something a half and any small piece a quarter.

The **decimal fraction** notation is widely used in everyday life, but many people do not understand decimal fractions thoroughly. This lack of understanding causes serious problems; for example, an inability to evaluate an answer obtained on a pocket calculator. **Percentages** are as widely used and even less well understood, especially when interest rates are involved. Many people are therefore at a disadvantage when they have to buy something on hire-purchase or lay-buy, to choose an insurance policy, or to borrow money.

Percentages are simply hundredths, and the percentage sign is simply another way of signifying that the denominator of the fraction is a hundred, e.g. 15% is another way of writing $\frac{15}{100}$.

Our decimal measuring system also uses fractions in a way that we are sometimes not aware of. A millimetre literally means a thousandth of a metre; a centimetre means a hundredth of a metre. (This is not the case in the imperial system used in South Africa up to about 1960 – there are three feet in a yard, but the word "foot" does not mean a third of a yard! An everyday exception to this is the quart, which is a quarter of a gallon.)

A good understanding of common fractions **forms the basis** for an understanding of decimal fractions, percentages and our decimal measure ment system. It is also necessary for algebra, algebraic manipulations, probability (chance) and statistics.

Children making sense of fraction situations

There is evidence of misconceptions of a particular kind, called limiting constructions, about fractions among primary school children. Some of these limiting constructions are caused by the child's pre-school or outside school experiences.

What is meant by limiting constructions? Individuals construct their own knowledge based on their own experiences. If their experiences only provide them with limited views of a particular concept, this **may close their minds** to the other aspects of the concept. We then say that these limited experiences have resulted in **limiting constructions**.

[1] General remarks on fractions re-printed with permission of the authors: Hanlie Murray and Amanda le Roux

For example, during the first few grades at school children only meet multiplication with whole numbers, and when you multiply two whole numbers (not 0 or 1), the answer is always bigger than either of the numbers. From this then develops the limiting construction that "multiplication makes bigger", which **severely hampers** children's understanding of how fractions behave.

Some limiting constructions can almost not be prevented (e.g. the above one), others **can definitely be prevented, by presenting children with problems that involve different experiences of and different views of a concept.**

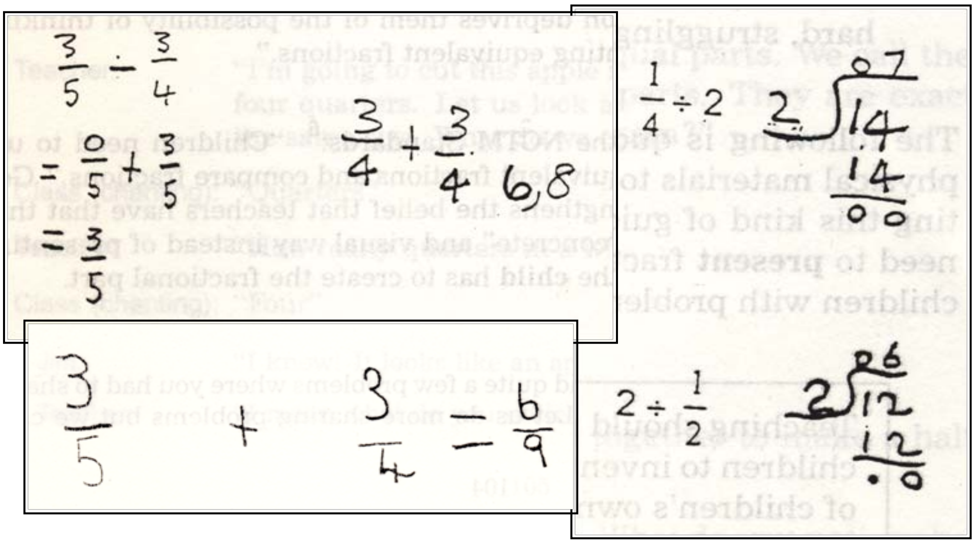
Limiting constructions

Research strongly indicates that teaching that gives a **limited vision on fractions** could cause many limiting constructions in children.

The following "mechanisms" that could inhibit the development of fractional knowledge were identified by researchers:

Whole number schemes:
These children perceive the symbol for a fraction (the fraction notation) to be made up by two whole numbers and they apply whole number strategies. This is clear in the following examples:

This limiting construction can be prevented by suspending the initial exposure to fraction notation for as long as possible.



Limited exposure to part-whole contexts:
This results in many children believing that a fraction is only a part of a single (continuous) whole, and then also only part of a circle or a square. They cannot then deal with a problem like sharing three pizzas among four friends or determining one third of a class of 27 children.

Knowledge of $\frac{1}{2}$:

The strategy of **repeated halving** to obtain other, smaller, fractions is so strong in some children that they find it difficult to imagine how something can be divided into thirds, fifths, sixths, etc. They can only imagine the sequence of halves, quarters, eighths, etc., for example by repeatedly folding a piece of paper.

Perceptual and visual representation:
Researchers find a major shortcoming in the teaching of equivalent fractions. Teachers use pictures and manipulatives when they introduce fractions, thus teaching it mainly perceptually and figuratively. The concepts formed by children stay figurative and they do not learn to **reason** about fractions.

An example to illustrate that a concept that was formed figuratively can cause a problem is given below. Children cannot see that halves of the rectangles below are the same even though they have different shapes.

Researchers also felt that ". . . by telling children that certain fractions are equivalent, traditional instruction deprives them of the possibility of thinking hard, struggling and inventing equivalent fractions."

The following is quoted from the NCTM Standards: "Children need to use physical materials to explore equivalent fractions and compare fractions." Getting this kind of guidance strengthens the belief that teachers have that they need to present fractions in a "concrete" and visual way instead of presenting children with problems where the **child has to create** the fractional part.

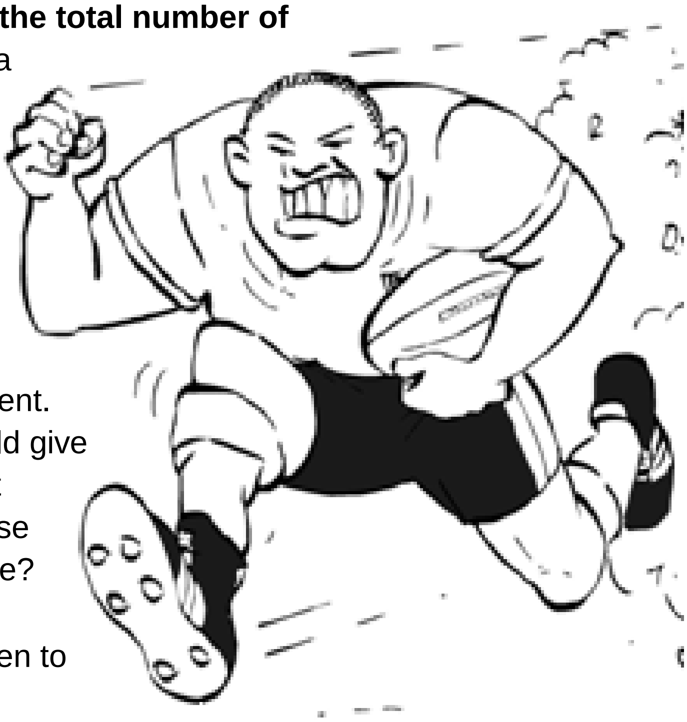
> Teaching should rather start with realistic problems that encourage children to invent their own solutions so that fractions can grow out of children's own thinking.

Let us review some reasons for problems with fractions.

1. The fraction concept is not well-developed and stable. It should be:
   - Anchored in word problems (situations) to allow children to reason about the situation
   - Developed over a long period, revisiting situations.

2. Special cases, like halves and quarters, are dealt with first. When learners are taught addition, they start with special cases where both fractions have the same denominator, followed by cases where the one denominator is a multiple of the other one. This often hides the real principles or reasons involved in a process.

3. Rules and techniques are given as social knowledge and they are required to memorise it, for example:
   - **Count the number of parts out of the total number of pieces**. This is given to children as a rule to find out what part of the whole a fraction is. Children should rather construct this knowledge when they engage in an equal sharing problem.
   - **Multiply the top and the bottom with the same number**. This is a rule to find fractions that are equivalent. Can you think of a problem that could give learners the opportunity to construct this knowledge so that it makes sense to them, instead of memorizing a rule?
   - **Multiply the tops and multiply the bottoms**. This rule is given to children to multiply two fractions.
   - **Multiply left top number with right bottom number, right top number with left bottom number; add; multiply bottom numbers**.
     or
     Divide smaller bottom number into bigger bottom number, multiply with top number of smaller bottom number, add top number of bigger bottom number. These are rules for addition.
   - **Invert the last one and multiply**. This is the rule for division by a fraction.



The fraction concept

Two sub-constructs: Even in its simplest form, there are two aspects to understand about a fractional part.

- Firstly, that it is one part of a number of equal parts into which a whole has been divided. This is referred to as the part-whole relationship between the fractional part and the unit, e.g. $\frac{1}{4}$ is one of four equal parts of a whole unit, and
- Secondly, that if this one part of the number of equal parts is repeated a certain fixed number of times, the whole is formed e.g. $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=1$ or, a quarter of a sausage means one of the four equal parts into which the whole sausage has been divided. But a quarter of a sausage also means that if it is taken 4 times, the whole sausage will be obtained.

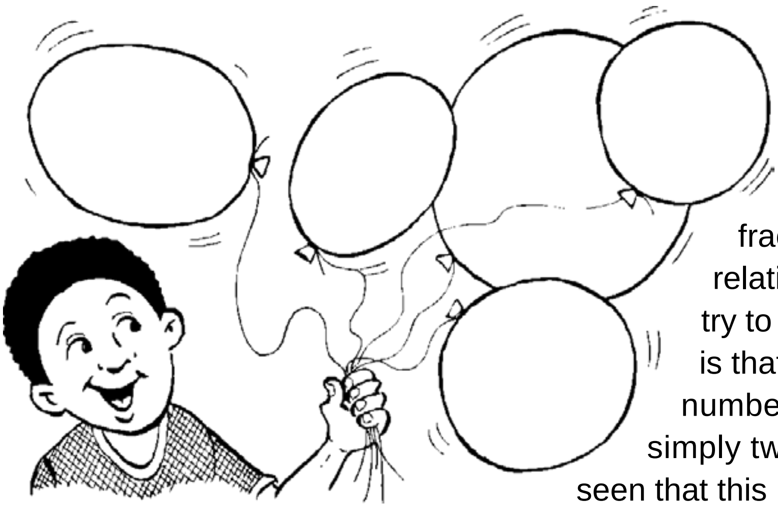
This sounds very simple and obvious, but many Grade 7 and 8 children are not clear about this second aspect.

Different meanings of fractions

Fractions are used in different ways to represent different things, depending on the situation. Fractions are used in different ways and with **different meanings**. Children should be exposed to these **different situations**, otherwise they cannot construct a full understanding of fractions.

Here are some of the meanings that fractions can have:

- A part of a whole, where the whole is a single object. e.g.:
  - one third of a sausage.
- A part of a whole, where the whole is a collection of objects. e.g.:
  - one third of the boys play soccer.
- A relationship or ratio or comparator. e.g.:
  - William earns a half of what his father earns.
  - A concrete mix suitable for paths is 1 part cement: 2 parts sand and 3 parts gravel.
- A unit of measurement. e.g.:
  - Three quarters of a metre. (Here the measuring unit is the "quarter metre.")
  - Ten centimetres wide (= 10 hundredths of a metre).
- A number. e.g.:
  - $3\frac{1}{4}$ is greater than 3 and less than 4.
  - Name two numbers between $4\frac{1}{2}$ and 5.
- A fraction can be used very abstractly as an operator. e.g.:
  - $\frac{3}{4}$ can simply imply "times 3 and divide by 4".
- The fraction notation is also used to represent division. e.g.:
  - $75 \div 25$ can be written as $\frac{75}{25}$



Any exercises or hints from the teacher that lead children to the idea that "you multiply by the top number and divide by the bottom number" can **interfere badly** with children's understanding of fractions as parts of wholes and as relationships and can hinder them when they try to solve problems. The main reason for this is that they may start regarding the two numbers that make up a common fraction as simply two separate whole numbers. We have seen that this is one of the serious limiting constructions in children's understanding of fractions that lead to many errors.

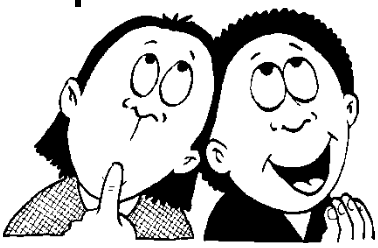
Teaching sequence for fractions

The fraction sequence that follows is taken from the ***Number Sense Workbook Series*** developed and distributed by Brombacher and Associates (www.brombacher.co.za).

Problem 1 Grade 2 Workbook 3 (page 19)

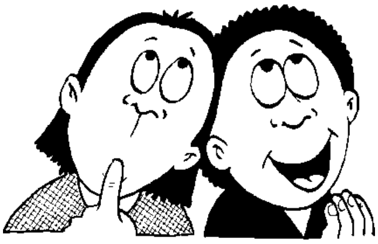
- Fundi and Yusuf want to share 3 chocolate bars equally. Show them how to do it.

- Jan, Sara and Ben want to share 4 chocolate bars equally. Show them how to do it.
- Yusuf, Ben, Jan and Fundi want to share 5 chocolate bars equally. Show them how to do it.



Problem 2 Grade 2 Workbook 3 (page 25)

- Three friends share 4 chocolate bars equally. Show them how to do it.
- Four friends share 5 chocolate bars equally. Show them how to do it.
- Five friends share 6 chocolate bars equally. Show them how to do it.



Problem 3 Grade 2 Workbook 3 (page 39)

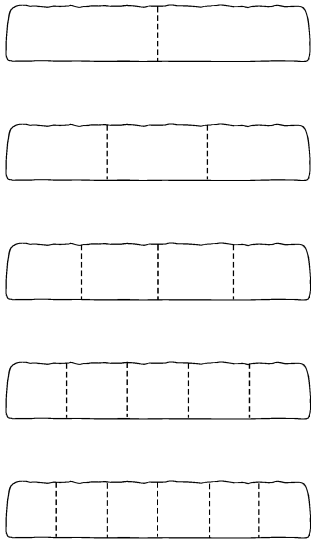
- Three friends share 7 chocolate bars equally. Show them how to do it.
- Four friends share 9 chocolate bars equally. Show them how to do it.

Introduction of social knowledge

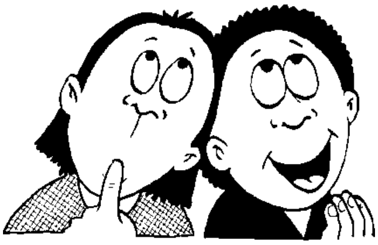
Problem 4 Grade 2 Workbook 3 (page 40)

If a chocolate bar is cut into:

- two equal parts, we call them halves
- three equal parts, we call them thirds
- four equal parts, we call them fourths
- five equal parts, we call them fifths
- six equal parts, we call them sixths

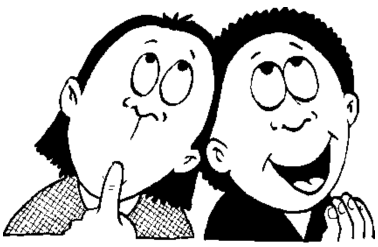


- Three friends share 4 chocolate bars equally. How much does each friend get?



Problem 5 Grade 2 Workbook 3 (page 46)

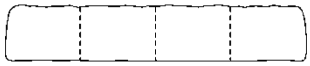
- Five friends share 6 chocolate bars equally. How much chocolate will each one get?
- Four friends share 9 chocolate bars equally. How much chocolate will each one get?
- Three friends share 7 chocolate bars equally. How much chocolate will each one get?
- What is bigger: one third of a chocolate bar or one fifth of a chocolate bar?



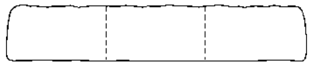
Problem 6 Grade 2 Workbook 4 (page 28)

These chocolate bars are cut into equal pieces

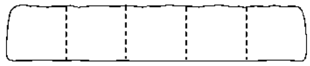
- Each piece is called ______________



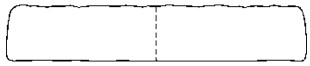
- Each piece is called ______________

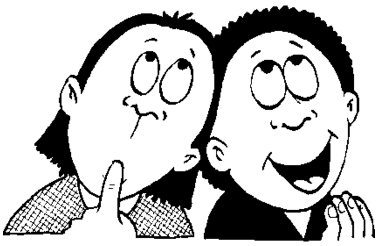


- Each piece is called ______________



- Each piece is called ______________





Problem 7 Grade 2 Workbook 4 (page 32)

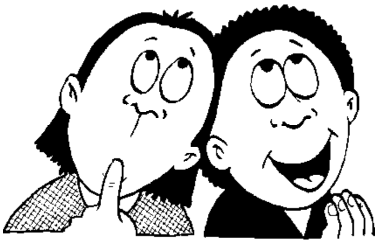
> Five friends share 6 vienna sausages equally. How much sausage will each one get? Show how they must do it.

Problem 8 Grade 2 Workbook 4 (page 40)

> Four friends share 9 sausages equally. How much sausage will each one get?

Problem 9 Grade 2 Workbook 4 (page 45)

> Three children share 10 chocolate bars equally. How much chocolate will each child get?



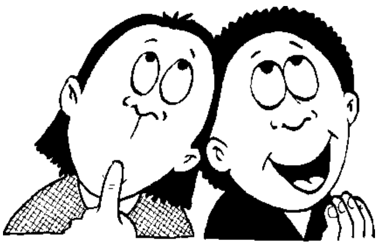
Sequence from Grade 3 booklet

Problem 1 Grade 3 Workbook 1 (page 27)

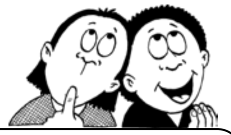
- Two friends want to share 3 chocolates bars equally. Show them how they can do it.
- Three friends want to share 4 chocolates bars equally. Show them how they can do it.

Problem 2 Grade 3 Workbook 1 (page 33)

- Four friends want to share 5 chocolate bars equally. Show how they can do it.
- Three friends want to share 7 chocolate bars equally. Show how they can do it.
- Four friends want to share 9 chocolate bars equally. Show how they can do it.

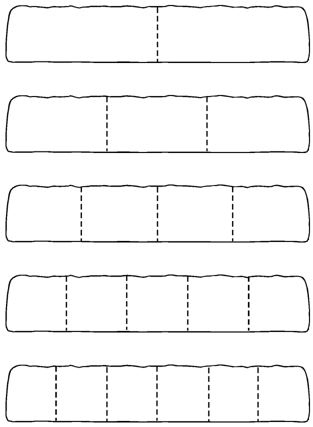


Problem 3 Grade 3 Workbook 1 (page 45)

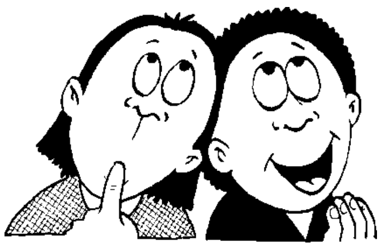


If a chocolate bar is cut into:

- two equal parts, we call them halves
- three equal parts, we call them thirds
- four equal parts, we call them fourths
- five equal parts, we call them fifths
- six equal parts, we call them sixths



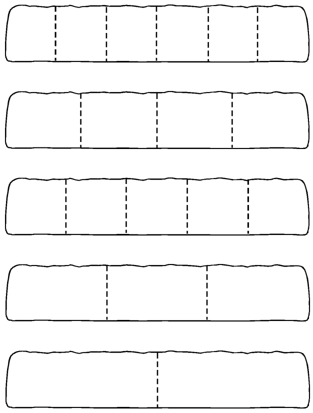
- Five friends share 6 chocolate bars equally. How much chocolate will each one get?
- Four friends share 9 chocolate bars equally. How much chocolate will each one get?
- Three friends share 7 chocolate bars equally. How much chocolate will each one get?



Problem 4 Grade 3 Workbook 1 (page 46)

These chocolate bars are cut into equal pieces.

- Each piece is called _____________
- Each piece is called _____________
- Each piece is called _____________
- Each piece is called _____________
- Each piece is called _____________



Problem 5 Grade 3 Workbook 2 (page 5)

- Three children share 10 chocolate bars equally. How much chocolate will each child get?

- Five children share 11 chocolate bars equally. How much chocolate will each child get?

Problem 6 Grade 3 Workbook 2 (page 14)

- Four children share 13 chocolate bars equally. How much chocolate will each child get?

- Six children share 7 chocolate bars equally. How much chocolate will each child get?

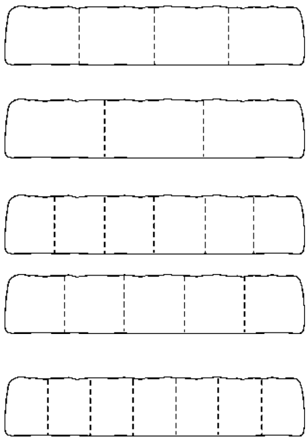
Problem 7 Grade 3 Workbook 2 (page 24)

- Three children share 5 chocolate bars equally. Show how they must do it.

- Four children share 6 chocolate bars equally. Show how they must do it.

Problem 8 Grade 3 Workbook 2 (page 26)

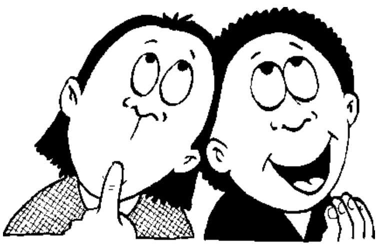
These chocolate bars are cut into equal pieces.



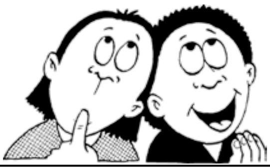
- Each piece is called _____________
- Each piece is called _____________
- Each piece is called _____________
- Each piece is called _____________
- Each piece is called _____________

Problem 9 Grade 3 Workbook 2 (page 27)

- Five friends share 11 vienna sausages equally. How much sausage will each get?
- Four friends share 17 vienna sausages equally. How much sausage will each get?



Problem 10 Grade 3 Workbook 2 (page 39)



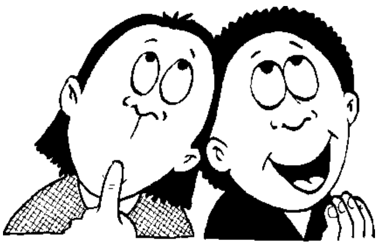
A short way of writing one half is $\frac{1}{2}$.

A short way of writing one third $\frac{1}{3}$.

A short way of writing one quarter $\frac{1}{4}$.

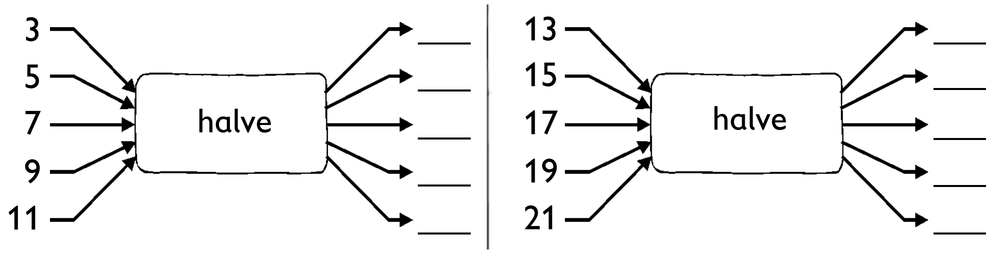
A short way of writing one fifth $\frac{1}{5}$.

- Three children share 7 chocolate bars equally. How much chocolate will each child get?

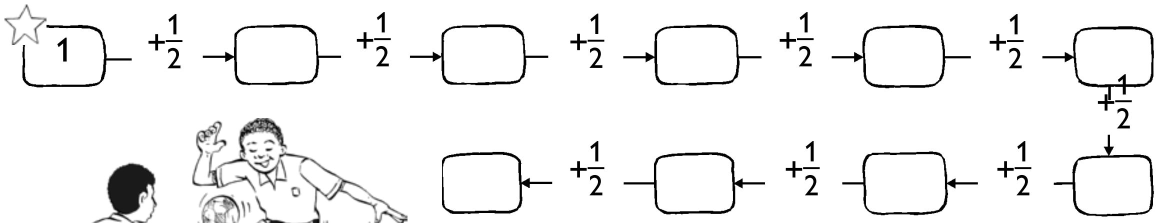


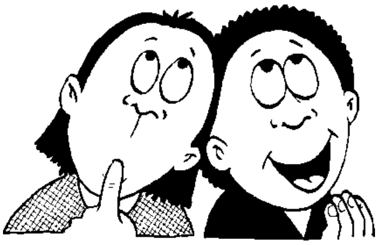
Problem 11 Grade 3 Workbook 2 (page 40)

Complete



Complete



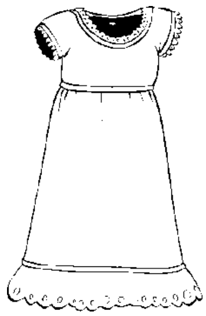


Problem 12 Grade 3 Workbook 2 (page 42)

One metre of ribbon costs 8c. Complete the table.

Length in metres	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$
Cost in cents	8		16		24			

Problem 13 Grade 3 Workbook 2 (page 43)



Mrs Twala has 20 metres of material. One dress uses $2\frac{1}{2}$ metres of material. Mrs Twala makes four dresses. How much material will she have left?

Problem 14 Grade 3 Workbook 2 (page 44)

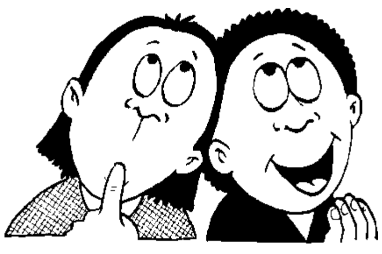
There are 60 minutes in one hour. Complete the table.

Hours	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$
Minutes	60		120		180			

Problem 15 Grade 3 Workbook 2 (page 44)

There are 60 minutes in one hour. Complete the table.

Hours	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$
Minutes	60		120		180			



Problem 16 Grade 3 Workbook 3 (page 3)

- Five friends share 12 chocolate bars equally. How must they do it?
- Five friends share 13 chocolate bars equally. How must they do it?

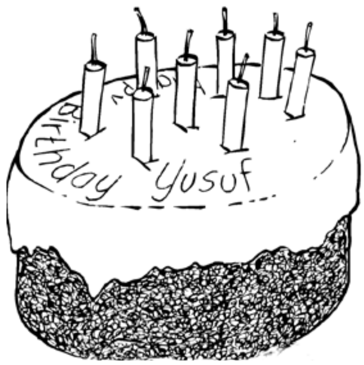
Problem 17 Grade 3 Workbook 3 (page 6)

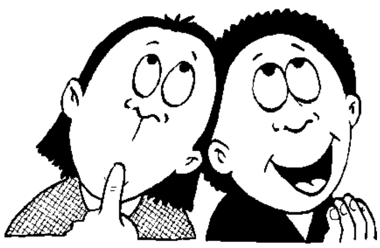
- 5 friends share 12 vienna sausages equally. How much sausage will each one get?
- 6 friends share 8 chocolates bars equally. How much chocolate will each one get?

Problem 18 Grade 3 Workbook 3 (page 9)

Help Ben work out what he needs to bake 3 cakes.

1 cake	**3 cakes**
1 egg	
$\frac{1}{2}$ cup oil	
3 cups flour	
$1\frac{1}{2}$ cups milk	
$1\frac{1}{4}$ cups sugar	
$2\frac{1}{2}$ teaspoons baking soda	

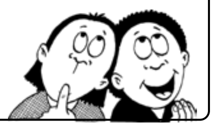




Problem 19 Grade 3 Workbook 3 (page 12)

A short way of writing one third is $\frac{1}{3}$.

A short way of writing two thirds is $\frac{2}{3}$.



Complete.

$\boxed{\frac{1}{3}} \xrightarrow{+\frac{1}{3}} \boxed{} \xrightarrow{+\frac{1}{3}} \boxed{}$

$\boxed{1\frac{1}{3}} \xrightarrow{+\frac{1}{3}} \boxed{} \xrightarrow{+\frac{1}{3}} \boxed{}$

$\boxed{2\frac{1}{3}} \xrightarrow{+\frac{1}{3}} \boxed{} \xrightarrow{+\frac{1}{3}} \boxed{}$

Problem 20 Grade 3 Workbook 3 (page 22)

Complete.

$\boxed{1\frac{1}{2}} \xrightarrow{+1\frac{1}{2}} \boxed{} \xrightarrow{+1\frac{1}{2}} \boxed{} \xrightarrow{+1\frac{1}{2}} \boxed{} \xrightarrow{+1\frac{1}{2}} \boxed{} \xrightarrow{+1\frac{1}{2}} \boxed{}$

$\downarrow +1\frac{1}{2}$

$\boxed{} \xleftarrow{+1\frac{1}{2}} \boxed{} \xleftarrow{+1\frac{1}{2}} \boxed{} \xleftarrow{+1\frac{1}{2}} \boxed{} \xleftarrow{+1\frac{1}{2}} \boxed{} \xleftarrow{+1\frac{1}{2}} \boxed{}$

$\downarrow +1\frac{1}{2}$

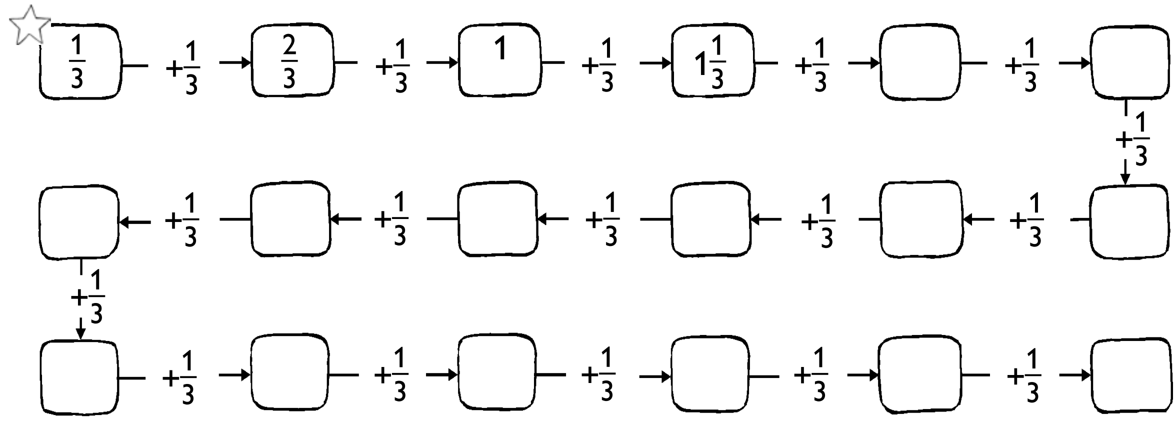
$\boxed{} \xrightarrow{+1\frac{1}{2}} \boxed{} \xrightarrow{+1\frac{1}{2}} \boxed{} \xrightarrow{+1\frac{1}{2}} \boxed{} \xrightarrow{+1\frac{1}{2}} \boxed{} \xrightarrow{+1\frac{1}{2}} \boxed{}$

One brick weighs $2\frac{1}{2}$ kilograms. Complete the table.

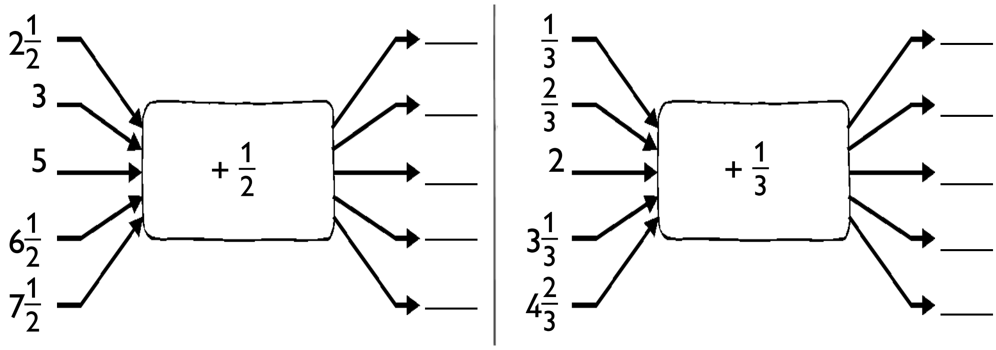
Bricks	1	2	3	4	5	6	8	10
kg	$2\frac{1}{2}$							

Problem 21 Grade 3 Workbook 3 (page 24)

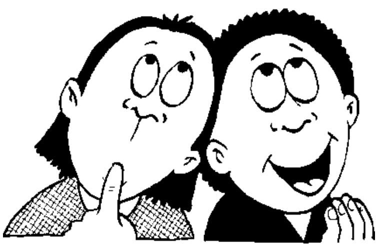
Complete.



Complete.



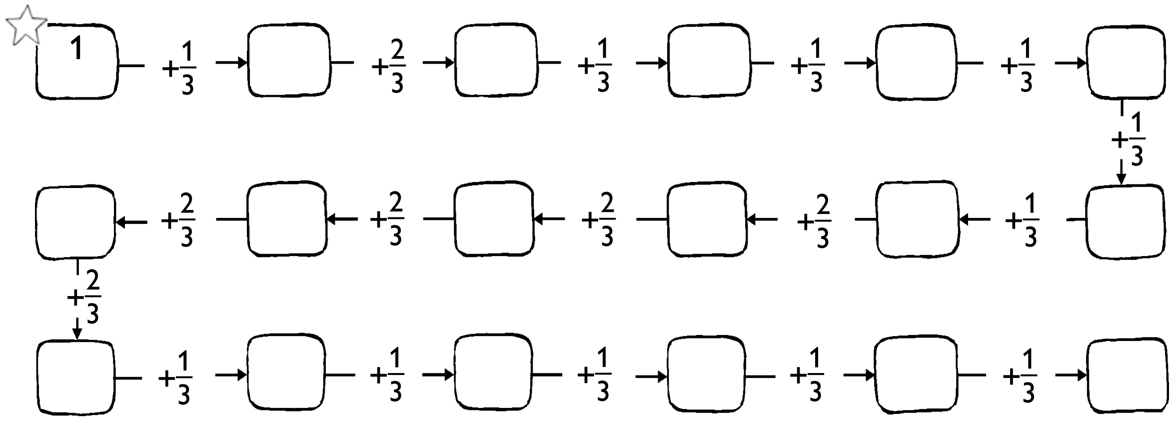
It takes $2\frac{1}{2}$ metres of material to make a dress. Fundi's mother wants to make 4 dresses. How much material does she need?



Problem 22 Grade 3 Workbook 3 (page 42) – same number chain as in problem 21

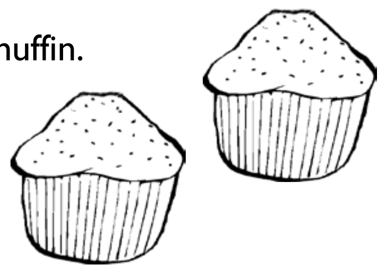
Problem 23 Grade 3 Workbook 4 (page 8)

Complete.



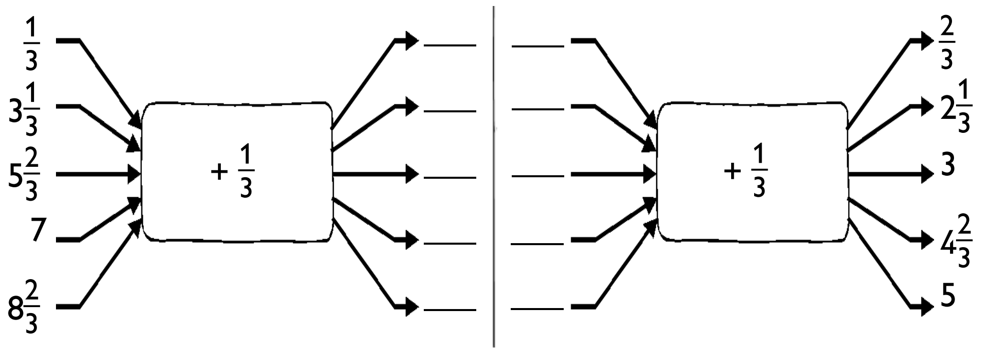
Mrs Faku needs $\frac{1}{3}$ of a cup of nuts to make one muffin.

She has 5 cups of nuts. How many muffins can she make?



Problem 24 Grade 3 Workbook 4 (page 10)

Complete.



Problem 25 Grade 3 Workbook 4 (page 12)

Complete the table.

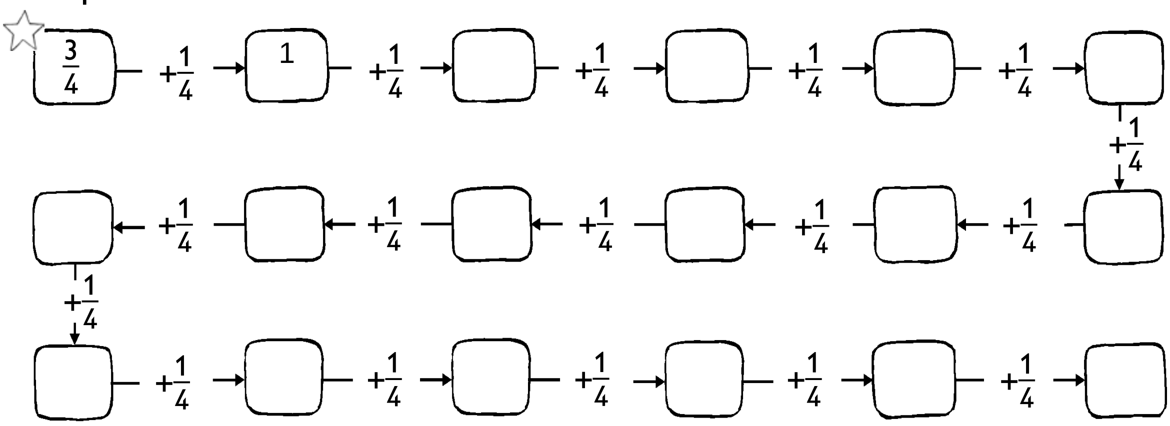
Hours	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1			3	4
Minutes				60	75	90		

Problem 26 Grade 3 Workbook 4 (page 20)

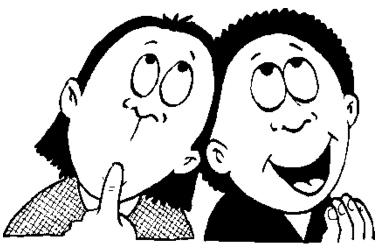
- 6 children share 8 chocolate bars equally. How much chocolate will each child get?
- 6 children share 9 chocolate bars equally. How much chocolate will each child get?

Problem 27 Grade 3 Workbook 4 (page 22)

Complete.



Sara needs $\frac{1}{4}$ of a metre of material to make one flag. How many flags can she make if she has 3 metres of material?



Problem 28 Grade 3 Workbook 4 (page 24)

Complete.

$\frac{4}{5}$ → $+\frac{1}{5}$ → 1 → $+\frac{1}{5}$ → ___ → $+\frac{1}{5}$ → ___ → $+\frac{1}{5}$ → ___ → $+\frac{1}{5}$ → ___ → $+\frac{1}{5}$ → ___

→ $+\frac{1}{5}$ → ___ → $+\frac{1}{5}$ → ___ → $+\frac{1}{5}$ → ___ → $+\frac{1}{5}$ → ___ → $+\frac{1}{5}$ → ___ → $+\frac{1}{5}$ → ___

→ $+\frac{1}{5}$ → ___ → $+\frac{1}{5}$ → ___ → $+\frac{1}{5}$ → ___ → $+\frac{1}{5}$ → ___ → $+\frac{1}{5}$ → ___ → $+\frac{1}{5}$ → ___

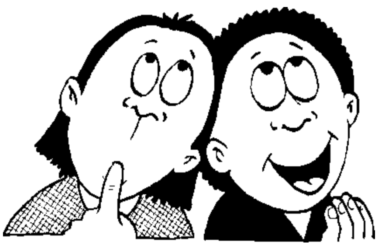
Complete.

	× 2	
13	→	___
14	→	___
___	→	29
___	→	30
___	→	31

	× 2	
$3\frac{1}{2}$	→	___
$7\frac{1}{2}$	→	___
$11\frac{1}{2}$	→	___
___	→	___
___	→	___

	× 2	
$\frac{1}{2}$	→	___
$\frac{1}{3}$	→	___
$\frac{1}{4}$	→	___
$\frac{1}{5}$	→	___
$\frac{2}{3}$	→	___

	× 2	
$1\frac{1}{2}$	→	___
$2\frac{1}{3}$	→	___
$3\frac{1}{4}$	→	___
$4\frac{1}{5}$	→	___
$1\frac{2}{3}$	→	___



Problem 29 Grade 3 Workbook 4 (page 31)

A steel pipe weighs $7\frac{1}{2}$ kilograms. Complete the tables.

Pipes	1	2	3	4	5	6	7	8
kg	$7\frac{1}{2}$	15						

Pipes	9	10	11	12	13	14	15	16
kg	$67\frac{1}{2}$	15						

Problem 30 Grade 3 Workbook 4 (page 37)

A brick weighs $2\frac{1}{2}$ kg. Complete the table.

Bricks	1	2	3	4	5	6	7	8
kg	$2\frac{1}{2}$	5						

Bricks	10	11	12	15	20			
kg						75	100	200