

**Brombacher
& Associates**

Developing the Fraction Concept

A workshop for Intermediate Phase teachers



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Three crucial factors

For the effective development of number sense and children's computational methods we need to pay attention to three crucial factors:

- Developing children's **understanding of number** through counting and manipulating numbers in various ways, measuring, number games, etc.;
- Using **meaningful problems** to introduce the basic operations and concepts and to support the development of increasingly sophisticated computational methods;
- Encouraging **discussion** by children to explain and compare their methods. It is only through discussion that children think about what they have done and try to understand what others have done. Through discussion they come to see the patterns, properties and relationships that underpin numeracy/mathematics and mathematical thinking.

Children must continuously be challenged to think, to reason and to make plans.

Different kinds of knowledge

The different types of knowledge (distinguished by Piaget) shed light on the processes by which the child learns about number – physical knowledge, social knowledge and logico-mathematical (conceptual) knowledge.

Physical knowledge

This is the kind of knowledge which children acquire through interaction with the physical world – e.g. through observing and handling objects

Physical knowledge is derived from concrete experiences – touching; using; playing with; and acting on concrete/physical material. Children need a lot of concrete experiences in the numeracy/mathematics classroom to develop their physical knowledge of number by counting concrete apparatus. It is through counting physical objects that children develop a sense of the size of numbers: 50 takes longer and more actions to count than 5 does but 250 takes a lot more. Five counters can be held in one hand; 50 in two hands; while 250 require a container – there are too many for our hands. Five counters look different from two counters.

Counting physical objects like counters is called rational counting – the counters are physically handled and moved from one place to another. The children observe the pile of counter grow as they count them.

Drawings are also objects in the sense that the counters described above are. Drawings are representations of the physical world.

The implication of physical knowledge for the Foundation Phase numeracy classroom is very simply that there must be both concrete apparatus (counters, shapes such as building blocks and other construction materials; and measuring apparatus) and the opportunity for children to work/play with the apparatus. It is the teacher's responsibility to provide the materials and the time for children to use them.

Social knowledge

The number five is an unproblematic concept for an adult who has known and used the word for many years; they can imagine five items and can even calculate with five without having to recreate the number using physical counters or representations in their minds. However, if we take a moment to reflect and think about this then we realise that the word five has no intrinsic properties that hint at the number of items it represents. So it is with people's names, place names, days of the week and months



of the year. The words we use to describe these are all “names” that we have assigned – and because the people in our community (society) all associate the same thing with the same name (word) we are able to communicate with each other. In order to know these names (this social knowledge) we need firstly to be told them and secondly to remember them.

We refer to knowledge that must be both told and remembered as social knowledge. The only way in which we can acquire this kind of knowledge is to be told it.

The implication of social knowledge for the classroom in general and the Foundation Phase classroom in particular is that teachers have to tell (teach) this to knowledge to children. They have to introduce children to the vocabulary.

The way in which we write the number symbols, indeed writing in general, is a socially agreed on conventions/habit and once more it is the job of the teacher in the Foundation Phase, in particular, to introduce children to these conventions.

In terms of numeracy/mathematics the words “addition, subtraction, division and multiplications” and the symbols that we use to denote them are all examples of social knowledge. Children “add” and “subtract” quite naturally without knowing the labels for the action that they are performing. When a mother ask her child to help her in the kitchen and says “I need 8 potatoes altogether, I already have three here, please get me the rest” then the child will respond to the question in one of two ways, either he/she will count on: “four, five, six, seven, eight” touching the potatoes while counting or he/she will count out eight potatoes from the packet and put three back. The first action could be called addition while the second subtraction. The point is that the child performs the actions without any knowledge of the word that the observer has to describe it.

Of concern, however, is the practice of many teachers of teaching as social knowledge the kinds of things that children can come to find out by themselves.

Conceptual (logico-mathematical) knowledge

When children reflect on activities (with or without physical objects) and begin to see patterns, relationships, regularities and irregularities within and between the numbers and the operations; they are constructing what is known as logico-mathematical knowledge. Logico-mathematical knowledge is internal knowledge and is constructed by each individual for themselves.

The teacher’s role in the development of children’s logico-mathematical knowledge is two-fold. On the one hand the teacher is responsible for creating activities and situations (problems) that will reveal the underlying structures of numbers, operations, and relationships. On the other hand the teacher needs to actively encourage children to reflect on what they are doing and what they are thinking – helping them to express these ideas in words so that they can explain their actions to others; discuss their respective methods; and even argue about the validity of each.

Since the teacher is unable to teach logico-mathematical knowledge through direct instruction (although it is tempting to try to do so – see the discussion of social knowledge above), one of the most important tasks of teaching is to design situations from which children can construct/develop their logico-mathematical knowledge. That is, the teacher needs to ask the question when designing a lesson/task: “What do I want children to learn from this situation/problem/activity?” Having established what it is that the teacher wants children to learn they then need to shape the situation/problem/activity in a way that

will provoke children to “see” the patterns and structures. Then, both during and on completion of the activity, the teacher needs to facilitate reflection on the activity by the child – it is this reflection more than anything else that will provoke the development of logico-mathematical knowledge.

Problem solving

In this section we deal with problem solving as well as the critical role of discussion.

In addressing this topic we will deal with the following questions:

- What is the role of problem solving in the learning of mathematics?
- What is meant by suitable problems? What types of problems should be posed?
- What is the role of discussion?

Children’s mathematical experiences should have a starting point in their own world.

What is the role of problem solving in the learning of mathematics?

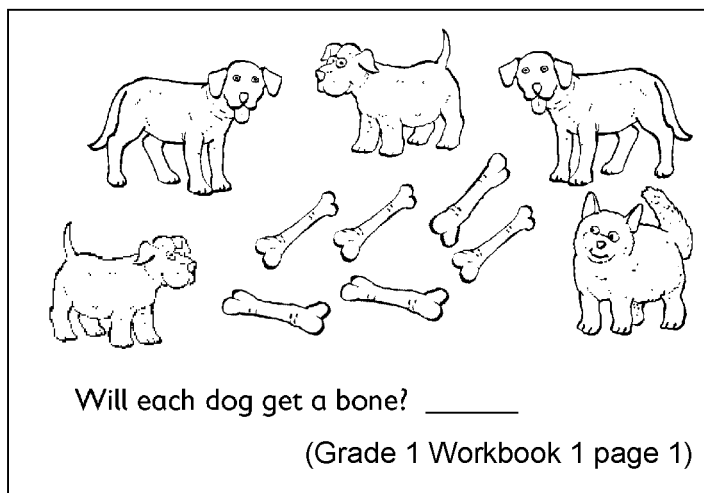
Problems are often thought of as a reason for studying mathematics. Mathematicians solve problems. In order to solve the problems that mathematicians solve they need to be able “to do” mathematics. That much is obvious – mathematics is the tool of the mathematician.

What is not always as well appreciated is the idea that problems also provide a way to introduce children to mathematics. Learning mathematics by solving problems! As much as this is true for mathematics at all levels, it is particularly true for children in the early years.

Consider for a moment a young child, say 4 years old. The child’s mother gives her some sweets with the instruction to share these with her little brother. Chances are good that she will be able to make a plan that will result in the fair/equal sharing of the sweets with her brother. Her strategy may well involve a “one for you, one for me, one for you, one for me, ...” dealing out of the sweets until the sweets are exhausted. In mathematical terms the girl has divided the sweets by two and the number of sweets that the girl and her brother each get can be determined by calculating the $(\text{number of sweets}) \div 2$. In mathematical terms the girl has completed a division problem and yet she may well be unable to count and may never have heard the word division!

Young children have a natural ability to solve problems and this problem solving ability can, and should, be used to good effect in the introduction of mathematics.

Not only does taking advantage of children’s natural problem solving ability provide a way of introducing mathematics and in particular the four basic operations (addition, subtraction, multiplication and division) of the Foundation Phase, but it also helps children to see value in the mathematics they are doing. Seeing value in what they are doing, in turn, gives children confidence and this confidence is important in helping children make a success of mathematics!



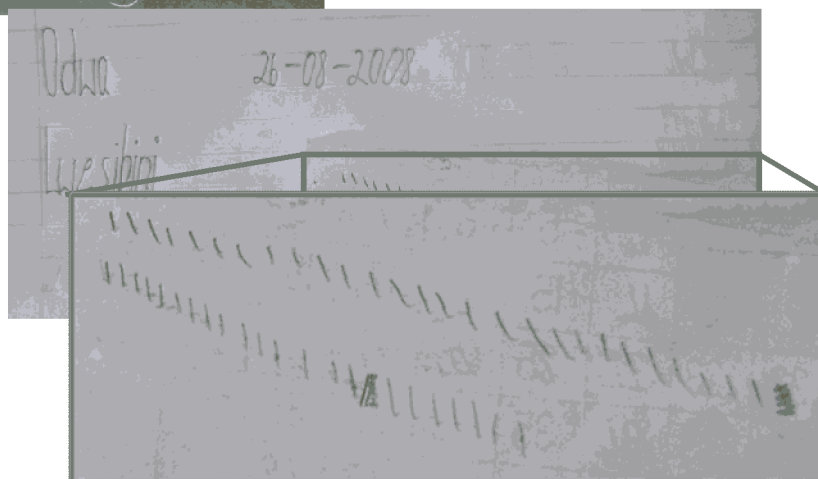
Consider another problem – the problem involving the dogs and the bones alongside. Children often solve this problem by simply drawing lines connecting the dogs and the bones, and in so doing

concluding that there are enough bones for each dog to get their own bone. Children can solve this problem without knowing number names, number symbols, the words addition and subtraction and, or for that matter, before they know the symbols representing these operations.

One more problem: Consider a problem used with children in a Foundation Phase class. The

Evenkileni banamagwinya
angama-27. Kodwa eklasini
kukho abantwana abangama-43

problem states: The tuckshop has made 27 amagwinya (vetkoek). There are 43 children in the class. Are there enough amagwinya for each child to get one? After an enthusiastic classroom discussion it was agreed that there were not enough amagwinya and the children were asked to determine how many more were needed.



Odwa solved the problem by first drawing 27 stripes to represent the 27 children in the class and then he drew a large number of extra stripes. He then counted on from 27: 28, 29, 30 ... 43 and highlighted the 43rd stripe. Finally he counted how many extra amagwinya (stripes) were needed and concluded that 16 more amagwinya needed to be made. Mathematically, we can summarise Odwa's solution as follows: $27 + 16 = 43$. That is, we can think of Odwa as having added on from 27 to 43 and in so doing to have established that 16 additional amagwinya were needed.

Asavelo solved the problem by first counting out 43 counters. Next she counted out 27 from the 43 – as if she was giving amagwinya to those whom she could give to. Finally she counted the remaining counters and established that she still needed 16 amagwinya for the remaining children. Mathematically, we can summarise Asavelo's solution as follows: $43 - 27 = 16$. That is, we can think of Asavelo as having subtracted 27 from 43 to establish that 16 additional amagwinya were needed.

The important point that this illustration makes is that the problem was not so much an addition problem or a subtraction problem, but rather a problem that could, in mathematical terms, be solved by means of both addition and subtraction. After a number of similar problems and appropriate discussion of both Odwa's and Asavelo's approaches the teacher's role becomes one of introducing the mathematical vocabulary and notation associated with these perfectly natural problem solving strategies.

Teachers can choose to either introduce a lesson (in the Foundation Phase) on addition and the rules



and methods associated with the procedure, or to present children with problems – problems that will

provoke the children to perform an action that is referred to as addition and then it remains for the teacher to introduce the vocabulary, symbols and conventions associated with the perfectly natural procedure.

There is one more observation to be made. Notice that both Odwa and Asavelo were confident about their answers. They could check them from the context. They had accounted for 43 children, they had used up the 27 amagwinya available and they could justify the need for an additional 16 amagwinya. For many children the only way they have of knowing whether or not their answer to the question: $43 - 27 = \square$ or $27 + \square = 43$ is correct, is to ask their teacher!

It goes without saying that as Odwa and Asavelo's number sense improves so they will rely less on stripes and counters and be able to use numbers with greater confidence in solving the same problems.

It is our strongly held conviction that children who learn mathematics through problems will see the value of what they are doing and be able to make sense of what they are doing far more so than those who don't.

Problems not only introduce children to the mathematical operations but problems also play an essential part in the development of computational methods

The structure of the problem initiates the first primitive method that the child constructs to solve that particular problem. As children solve problems over a long period of time, their number concept develops and their methods are refined. With time then develop a more integrated understanding of the operations and the strategy they use will become more sophisticated.

What is the role of discussion?

Interaction about what children have done is important while learning mathematics, because it stimulates reflection and enables them to function at a higher level.

In order to optimize the role of discussion in teaching and learning, a specific classroom culture should be established. This could be seen as a didactical contract, where teachers and learners communicate and negotiate their expectations and obligations.

- The learners must know that they may use any method to solve a problem, but they are expected to explain their ideas and methods to the teacher and to the others in the group or class. They are also expected to listen to the explanations of others in their group or class. Only an explanation that is clear and understood should be accepted. The way in which the teacher handles (and accepts or does not accept) an explanation, indicates that a good, acceptable explanation is one that is understood by everybody in the group or class.
- Learners soon learn to listen critically to one another's explanations and challenge it when it is not understood or clear. This type of interaction (argumentative) is the most powerful for learning to take place. The teacher facilitates guides and gives structure to the interaction and makes the ideas that come up in the discussion explicit for all the learners in the class.
- Because children are expected to make sense of mathematics by working on and solving problems, it is essential that the mathematical ideas which they construct during the process are put "in public". This makes it possible for the teacher to detect any possible errors in thinking or misconceptions that may exist in the child's mind. It also gives the teacher the opportunity to check whether the child uses the conventions of mathematics and terminology correctly when communicating about mathematics and recording their methods.

Fractions – general remarks¹

The **common fraction** notation for fractions is used less frequently in everyday life than it was before; as a result children are not as used to it as they are to whole numbers. When common fractions are used, it is mostly in an inexact way - young children may therefore call any big piece of something a half and any small piece a quarter.

The **decimal fraction** notation is widely used in everyday life, but many people do not understand decimal fractions thoroughly. This lack of understanding causes serious problems; for example, an inability to evaluate an answer obtained on a pocket calculator. **Percentages** are as widely used and even less well understood, especially when interest rates are involved. Many people are therefore at a disadvantage when they have to buy something on hire-purchase or lay-buy, to choose an insurance policy, or to borrow money.

Percentages are simply hundredths, and the percentage sign is simply another way of signifying that the denominator of the fraction is a hundred, e.g. 15% is another way of writing $\frac{15}{100}$.

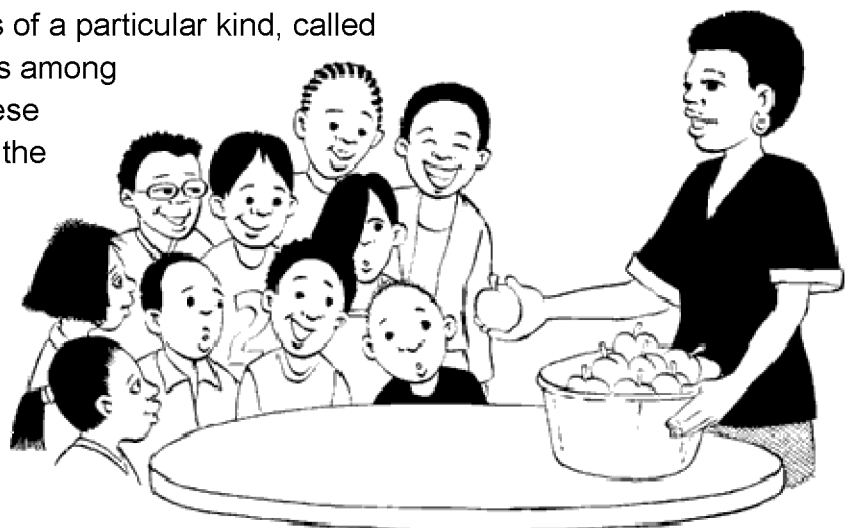
Our decimal measuring system also uses fractions in a way that we are sometimes not aware of. A millimetre literally means a thousandth of a metre; a centimetre means a hundredth of a metre. (This is not the case in the imperial system used in South Africa up to about 1960 – there are three feet in a yard, but the word "foot" does not mean a third of a yard! An everyday exception to this is the quart, which is a quarter of a gallon.)

A good understanding of common fractions **forms the basis** for an understanding of decimal fractions, percentages and our decimal measurement system. It is also necessary for algebra, algebraic manipulations, probability (chance) and statistics.

Children making sense of fraction situations

There is evidence of misconceptions of a particular kind, called limiting constructions, about fractions among primary school children. Some of these limiting constructions are caused by the child's pre-school or outside school experiences.

What is meant by limiting constructions? Individuals construct their own knowledge based on their own experiences. If their experiences only provide them with limited views of a particular concept, this **may close their minds** to the other aspects of the concept. We then say that these limited experiences have resulted in **limiting constructions**.



¹ General remarks on fractions re-printed with permission of the authors: Hanlie Murray and Amanda le Roux

For example, during the first few grades at school children only meet multiplication with whole numbers, and when you multiply two whole numbers (not 0 or 1), the answer is always bigger than either of the numbers. From this then develops the limiting construction that "multiplication makes bigger", which **severely hampers** children's understanding of how fractions behave.

Some limiting constructions can almost not be prevented (e.g. the above one), others **can definitely be prevented, by presenting children with problems that involve different experiences of and different views of a concept.**

Limiting constructions

Research strongly indicates that teaching that gives a **limited vision on fractions** could cause many limiting constructions in children.

The following "mechanisms" that could inhibit the development of fractional knowledge were identified by researchers:

Whole number schemes:

These children perceive the symbol for a fraction (the fraction notation) to be made up by two whole numbers and they apply whole number strategies. This is clear in the following examples:

This limiting construction can be prevented by suspending the initial exposure to fraction notation for long as possible.

The image shows three examples of handwritten student work illustrating whole number schemes:

- Top left: $\frac{3}{5} \div \frac{3}{4}$ is incorrectly calculated as $\frac{0}{5} + \frac{3}{5} = \frac{3}{5}$.
- Top right: $\frac{1}{4} \div 2$ is incorrectly calculated as $\frac{3}{4} + \frac{3}{4} = 6,8$.
- Bottom left: $\frac{3}{5} + \frac{3}{4} = \frac{6}{9}$.
- Bottom right: $2 \div \frac{1}{2}$ is incorrectly calculated as $2 \times 6 = 12$.

as

Limited exposure to part-whole contexts:

This results in many children believing that a fraction is only a part of a single (continuous) whole, and then also only part of a circle or a square. They cannot then deal with a problem like sharing three pizzas among four friends or determining one third of a class of 27 children.

Knowledge of $\frac{1}{2}$:

The strategy of **repeated halving** to obtain other, smaller, fractions is so strong in some children that they find it difficult to imagine how something can be divided into thirds, fifths, sixths, etc. They can only imagine the sequence of halves, quarters, eighths, etc., for example by repeatedly folding a piece of paper.

Perceptual and visual representation:

Researchers find a major shortcoming in the teaching of equivalent fractions. Teachers use pictures and manipulatives when they introduce fractions, thus teaching it mainly perceptually and figuratively. The concepts formed by children stay figurative and they do not learn to **reason** about fractions.

An example to illustrate that a concept that was formed figuratively can cause a problem is given below. Children cannot see that halves of the rectangles below are the same even though they have different shapes.



Researchers also felt that ". . . by telling children that certain fractions are equivalent, traditional instruction deprives them of the possibility of thinking hard, struggling and inventing equivalent fractions."

The following is quoted from the NCTM Standards: "Children need to use physical materials to explore equivalent fractions and compare fractions." Getting this kind of guidance strengthens the belief that teachers have that they need to present fractions in a "concrete" and visual way instead of presenting children with problems where the **child has to create** the fractional part.

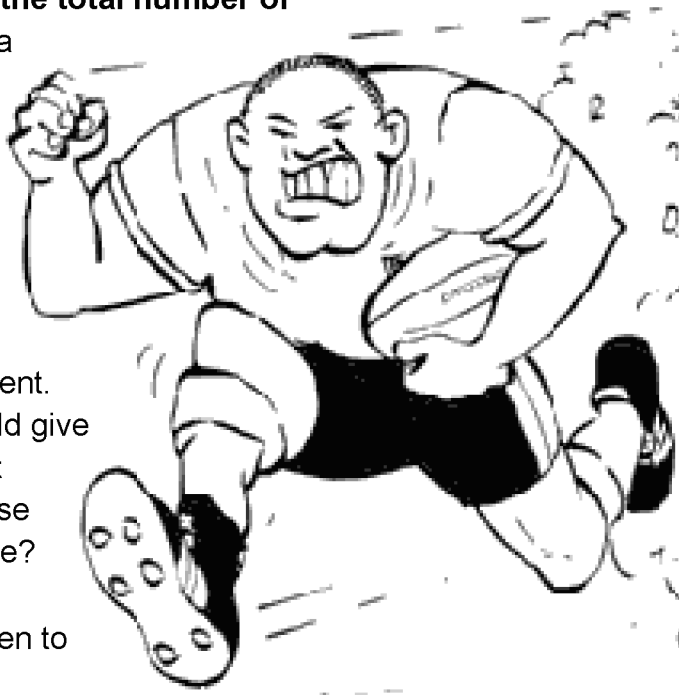
Teaching should rather start with realistic problems that encourage children to invent their own solutions so that fractions can grow out of children's own thinking.

Let us review some reasons for problems with fractions.

1. The fraction concept is not well-developed and stable. It should be:
 - Anchored in word problems (situations) to allow children to reason about the situation
 - Developed over a long period, revisiting situations.
2. Special cases, like halves and quarters, are dealt with first. When learners are taught addition, they start with special cases where both fractions have the same denominator, followed by cases where the one denominator is a multiple of the other one. This often hides the real principles or reasons involved in a process.

3. Rules and techniques are given as social knowledge and they are required to memorise it, for example:

- **Count the number of parts out of the total number of pieces.** This is given to children as a rule to find out what part of the whole a fraction is. Children should rather construct this knowledge when they engage in an equal sharing problem.
- **Multiply the top and the bottom with the same number.** This is a rule to find fractions that are equivalent. Can you think of a problem that could give learners the opportunity to construct this knowledge so that it makes sense to them, instead of memorizing a rule?
- **Multiply the tops and multiply the bottoms.** This rule is given to children to multiply two fractions.
- **Multiply left top number with right bottom number, right top number with left bottom number; add; multiply bottom numbers.**
or
Divide smaller bottom number into bigger bottom number, multiply with top number of smaller bottom number, add top number of bigger bottom number. These are rules for addition.
- **Invert the last one and multiply.** This is the rule for division by a fraction.



The fraction concept

Two sub-constructs: Even in its simplest form, there are two aspects to understand about a fractional part.

- Firstly, that it is one part of a number of equal parts into which a whole has been divided. This is referred to as the part-whole relationship between the fractional part and the unit, e.g. $\frac{1}{4}$ is one of four equal parts of a whole unit, and
- Secondly, that if this one part of the number of equal parts is repeated a certain fixed number of times, the whole is formed e.g. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1$ or, a quarter of a sausage means one of the four equal parts into which the whole sausage has been divided. But a quarter of a sausage also means that if it is taken 4 times, the whole sausage will be obtained.

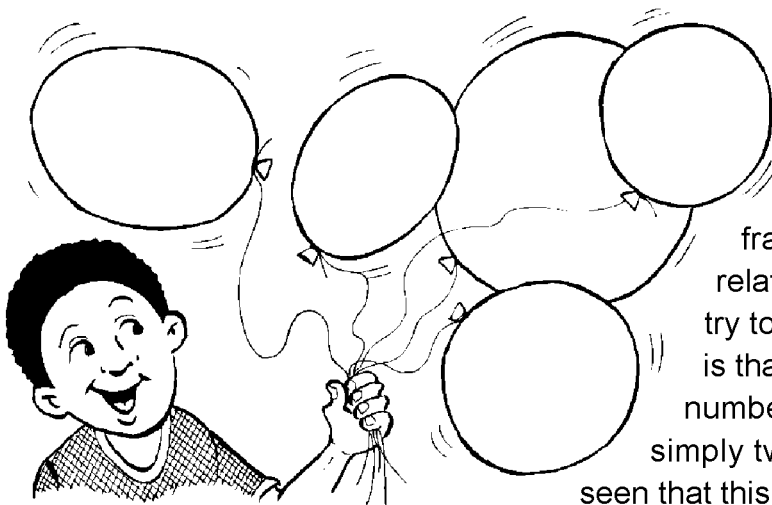
This sounds very simple and obvious, but many Grade 7 and 8 children are not clear about this second aspect.

Different meanings of fractions

Fractions are used in different ways to represent different things, depending on the situation. Fractions are used in different ways and with **different meanings**. Children should be exposed to these **different situations**, otherwise they cannot construct a full understanding of fractions.

Here are some of the meanings that fractions can have:

- A part of a whole, where the whole is a single object. e.g.:
 - one third of a sausage.
- A part of a whole, where the whole is a collection of objects. e.g.:
 - one third of the boys play soccer.
- A relationship or ratio or comparator. e.g.:
 - William earns a half of what his father earns.
 - A concrete mix suitable for paths is 1 part cement: 2 parts sand and 3 parts gravel.
- A unit of measurement. e.g.:
 - Three quarters of a metre. (Here the measuring unit is the "quarter metre.")
 - Ten centimetres wide (= 10 hundredths of a metre).
- A number. e.g.:
 - $3\frac{1}{4}$ is greater than 3 and less than 4.
 - Name two numbers between $4\frac{1}{2}$ and 5.
- A fraction can be used very abstractly as an operator. e.g.:
 - $\frac{3}{4}$ can simply imply "times 3 and divide by 4".
- The fraction notation is also used to represent division. e.g.:
 - $75 \div 25$ can be written as $\frac{75}{25}$



Any exercises or hints from the teacher that lead children to the idea that "you multiply by the top number and divide by the bottom number" can **interfere badly** with children's understanding of fractions as parts of wholes and as relationships and can hinder them when they try to solve problems. The main reason for this is that they may start regarding the two numbers that make up a common fraction as simply two separate whole numbers. We have seen that this is one of the serious limiting

constructions in children's understanding of fractions that lead to many errors.

The big ideas that underpin the development of the fraction concept

The big ideas that underpin a robust sense of fraction include:

- Fraction concept
- Equivalence
- Addition and subtraction
- Multiplication
- Division

The aspects that are related to each of the “big ideas” are listed below. Each of these aspects is developed over a **long period of time** and should often be **revisited**.

The basic fraction concept:

To develop a sound concept of a fraction (as part of a whole), learners should have the following experiences with:

- Equal sharing situations (dividing the whole into parts) where the need for a unitary fraction is created,
- Noticing that the whole can be cut into any number of parts (parts that become known as fractions).
- Exposure to the names of fractions (given as social knowledge).
- Determining the name of a given part in a diagram, or the result of a sharing situation (assessment of knowledge of fraction names – children have to know how to name a fraction).
- Comparing the relative sizes of fractions which are the result of equal sharing situations (this should be done quite early – allowing children to think about the relative size of fractions, when the first equal sharing is done)
- Equal sharing situations that result in non-unitary fractions.
- Fraction notation (given as social knowledge).
- Combining fractions to form wholes.
- “Counting” in fractions (as in fraction chains) – developing a “number concept” regarding fractions – this allows children to use the fraction notation,
- Grouping and reasoning about groups
- Determining a fraction of a collection (in a context).
- Determining a fraction of a number (without a context).
- Repeated addition of fractions context and without context.
- Situations that give rise to the existence of the equivalence of fractions.
- Placing fractions on a number line – completing no lines and estimating positions.
- The “inverse” of finding a fraction of a collection – e.g. one-fifth of the class is 7 – how many in the class? (this is quite difficult – hence late on the list.)
- Finding a fraction of a fraction

For a comprehensive/complete fraction concept, learners should solve a variety of word problems involving fractions in informal ways – these include repeated addition, division (grouping – reasoning about how many... in ... ?), fraction of a collection and fraction of a fraction.

These experiences prepare learners for the development of equivalence and performing the basic operations with fractions formally – eventually understanding the various algorithms.

Equivalence

The following pre-knowledge is necessary for understanding equivalence:

- How to name a fraction:
 - A whole can be cut into any number of equal parts to form fractions. The name of a fraction depends on the number of equal parts.
 - Awareness that parts of different sizes can have different names
- Awareness of the existence of equivalence. This can be introduced through a problem that leads to equal sharing in more than one way: e.g. 8 shared by 6 gives rise to 1 and two-sixths or 1 and one-third.

The concept of equivalence can be developed by means of the following types of problems:

- Fraction of a fraction
e.g. finding a half of a third – a third is cut into 2 equal parts. These new parts are called sixths and it is evident that a third is the same as 2 sixths.
- Fraction of a collection
In these examples fractions are expressed in terms of whole numbers. This makes it possible to compare the relative sizes:
 - $\frac{1}{3}$ of 18 smarties = $\frac{2}{6}$ of 18 smarties
 - $\frac{2}{5}$ of 60 minutes = $\frac{4}{10}$ of 60 minutes

The following must become explicit when equivalence is being developed:

- A fraction can only be expressed in terms of another fraction, if the denominator of the second fraction is a multiple (or sometimes a factor) of the first fraction. “Fraction of fraction” problems are suitable for producing “families of fractions”:

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15} = \frac{6}{18}$$

Operations with fractions

The types of fraction problems that help to develop each of the operations with fractions are listed below. Each problem type should be given in a context as well as without a context (as a “naked” calculation).

Addition and subtraction

- Combining parts to form wholes
- Repeated addition of same fraction
- Awareness of equivalence
- Being able to form equivalent fractions
- Awareness of fact that quantities in different units cannot be added

Multiplication

- Repeated addition
- Determining a fraction of a collection

- Determining a fraction of a fraction
- Multiplication of fractions is eventually defined as “of” – the problems in the previous bullets are then seen as multiplication
- Equivalence

Division

- How many ... in ...?
- Repeated addition
- Equivalence

Teaching sequence for fractions

The fraction sequence that follows is taken from the **Number Sense Workbook Series** developed and distributed by Brombacher and Associates (www.brombacher.co.za).

Sequence from Grade 3 booklets

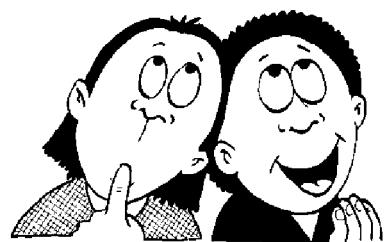
Problem 1 (Number Sense Workbook 9, page 27)

- Two friends want to share 3 chocolate bars equally. Show them how they can do it.
- Three friends want to share 4 chocolate bars equally. Show them how they can do it.



Problem 2 (Number Sense Workbook 9, page 33)

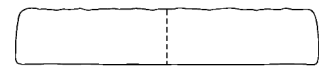
- Four friends want to share 5 chocolate bars equally. Show how they can do it.
- Three friends want to share 7 chocolate bars equally. Show how they can do it.
- Four friends want to share 9 chocolate bars equally. Show how they can do it.



Problem 3 (Number Sense Workbook 9, page 45)

If a chocolate bar is cut into:

- two equal parts, we call them halves
- three equal parts, we call them thirds
- four equal parts, we call them fourths
- five equal parts, we call them fifths
- six equal parts, we call them sixths

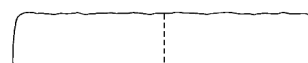
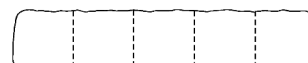
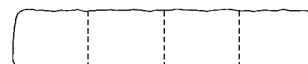


- Five friends share 6 chocolate bars equally. How much chocolate will each one get?
- Four friends share 9 chocolate bars equally. How much chocolate will each one get?
- Three friends share 7 chocolate bars equally. How much chocolate will each one get?

Problem 4 (Number Sense Workbook 9, page 46)

These chocolate bars are cut into equal pieces.

- Each piece is called _____
- Each piece is called _____
- Each piece is called _____
- Each piece is called _____
- Each piece is called _____



Problem 5 (Number Sense Workbook 10, page 5)

- Three children share 10 chocolate bars equally. How much chocolate will each child get?
- Five children share 11 chocolate bars equally. How much chocolate will each child get?

Problem 6 (Number Sense Workbook 10, page 14)

- Four children share 13 chocolate bars equally. How much chocolate will each child get?
- Six children share 7 chocolate bars equally. How much chocolate will each child get?

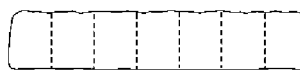
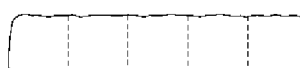
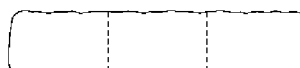
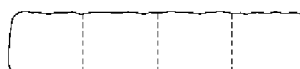
Problem 7 (Number Sense Workbook 10, page 24)

- Three children share 5 chocolate bars equally. Show how they must do it.
- Four children share 6 chocolate bars equally. Show how they must do it.

Problem 8 (Number Sense Workbook 10, page 2)

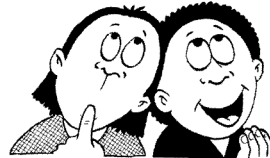
These chocolate bars are cut into equal pieces.

- Each piece is called _____
- Each piece is called _____
- Each piece is called _____
- Each piece is called _____
- Each piece is called _____



Problem 9 (Number Sense Workbook 10, page 27)

- Five friends share 11 vienna sausages equally. How much sausage will each get?
- Four friends share 17 vienna sausages equally. How much sausage will each get?

Problem 10 (Number Sense Workbook 10, page 39)

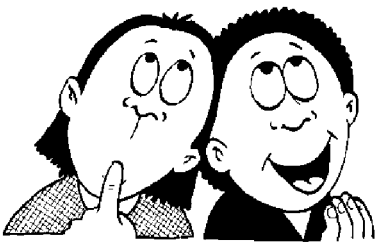
A short way of writing one half is $\frac{1}{2}$.

A short way of writing one third $\frac{1}{3}$.

A short way of writing one quarter $\frac{1}{4}$.

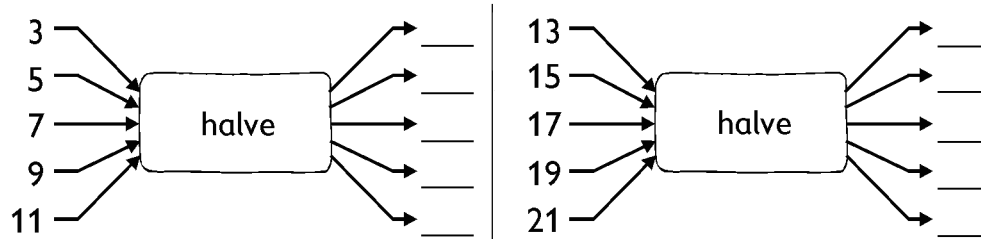
A short way of writing one fifth $\frac{1}{5}$.

- Three children share 7 chocolate bars equally. How much chocolate will each child get?

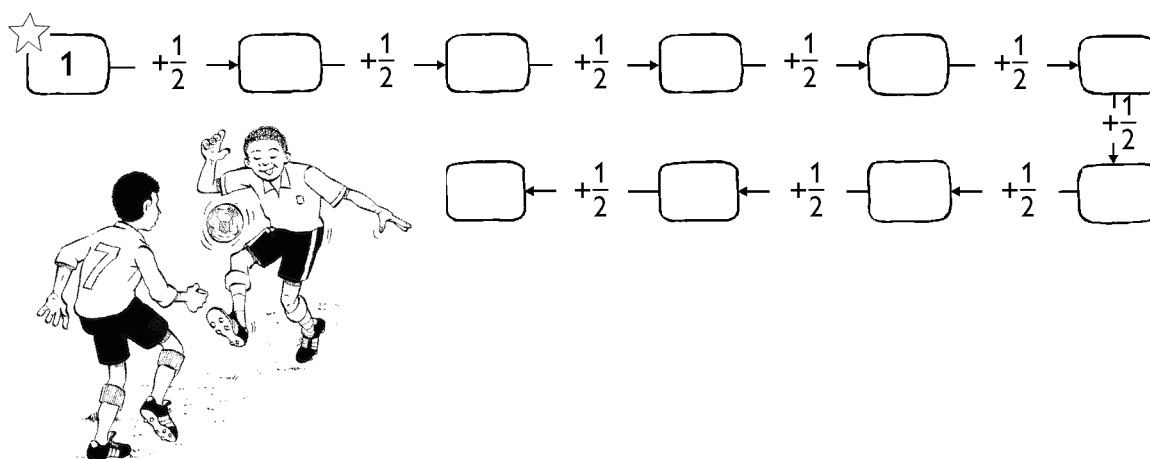


Problem 11 (Number Sense Workbook 10, page 40)

Complete



Complete

**Problem 12** (Number Sense Workbook 10, page 42)

One metre of ribbon costs 8c. Complete the table.

Length in metres	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$
Cost in cents	8		16		24			

Problem 13 (Number Sense Workbook 10, page 43)

Mrs Twala has 20 metres of material. One dress uses $2\frac{1}{2}$ metres of material. Mrs Twala makes four dresses. How much material will she have left?



Problem 14 (Number Sense Workbook 10, page 44)

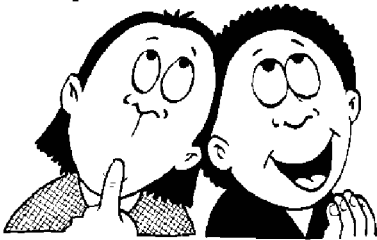
There are 60 minutes in one hour. Complete the table.

Hours	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$
Minutes	60		120		180			

Problem 15 (Number Sense Workbook 10, page 4)

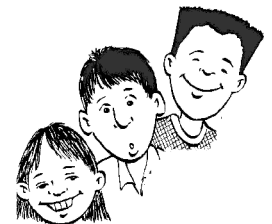
There are 60 minutes in one hour. Complete the table.

Hours	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$
Minutes	60		120		180			



Problem 16 (Number Sense Workbook 11, page 3)

- Five friends share 12 chocolate bars equally. How must they do it?
- Five friends share 13 chocolate bars equally. How must they do it?

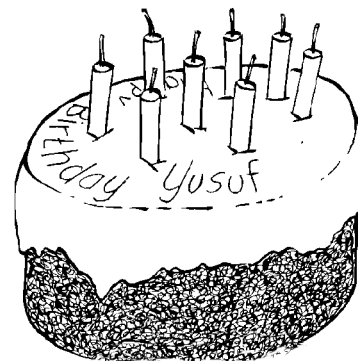
Problem 17 (Number Sense Workbook 11, page 6)

- 5 friends share 12 vienna sausages equally. How much sausage will each one get?
- 6 friends share 8 chocolates bars equally. How much chocolate will each one get?

Problem 18 (Number Sense Workbook 11, page 9)

Help Ben work out what he needs to bake 3 cakes.

1 cake	3 cakes
1 egg	
$\frac{1}{2}$ cup oil	
3 cups flour	
$1\frac{1}{2}$ cups milk	
$1\frac{1}{4}$ cups sugar	
$2\frac{1}{2}$ teaspoons baking soda	

Problem 19 (Number Sense Workbook 11, page 12)

A short way of writing one third is $\frac{1}{3}$.

A short way of writing two thirds is $\frac{2}{3}$.

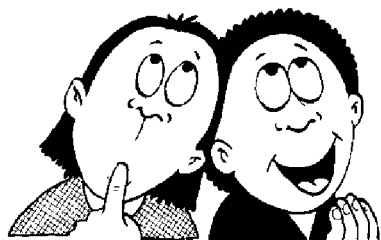


Complete.

$$\boxed{\frac{1}{3}} - \frac{1}{3} \rightarrow \boxed{} - \frac{1}{3} \rightarrow \boxed{}$$

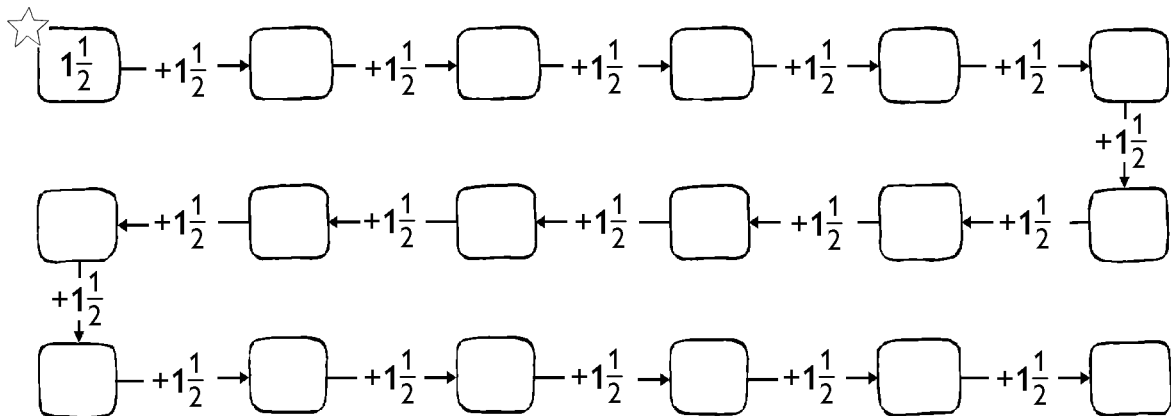
$$\boxed{1\frac{1}{3}} - \frac{1}{3} \rightarrow \boxed{} - \frac{1}{3} \rightarrow \boxed{}$$

$$\boxed{2\frac{1}{3}} - \frac{1}{3} \rightarrow \boxed{} - \frac{1}{3} \rightarrow \boxed{}$$



Problem 20 (Number Sense Workbook 11, page 22)

Complete.

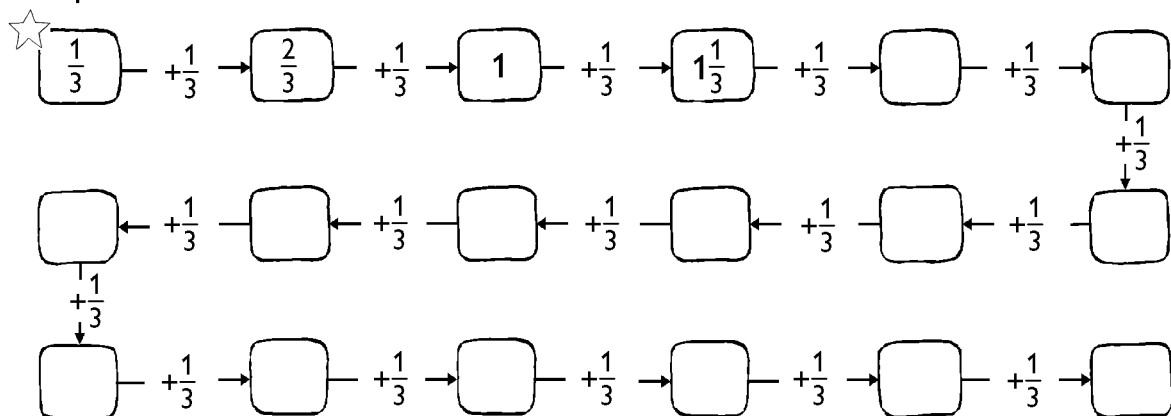


One brick weighs $2\frac{1}{2}$ kilograms. Complete the table.

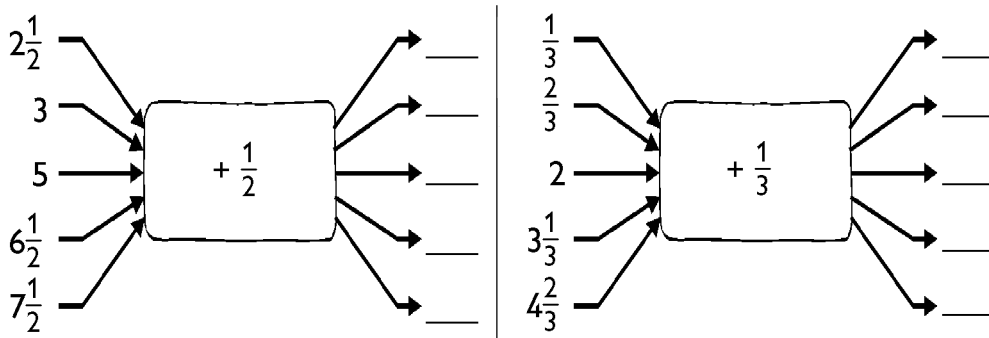
Bricks	1	2	3	4	5	6	8	10
kg	$2\frac{1}{2}$							

Problem 21 (Number Sense Workbook 11, page 24)

Complete.



Complete.

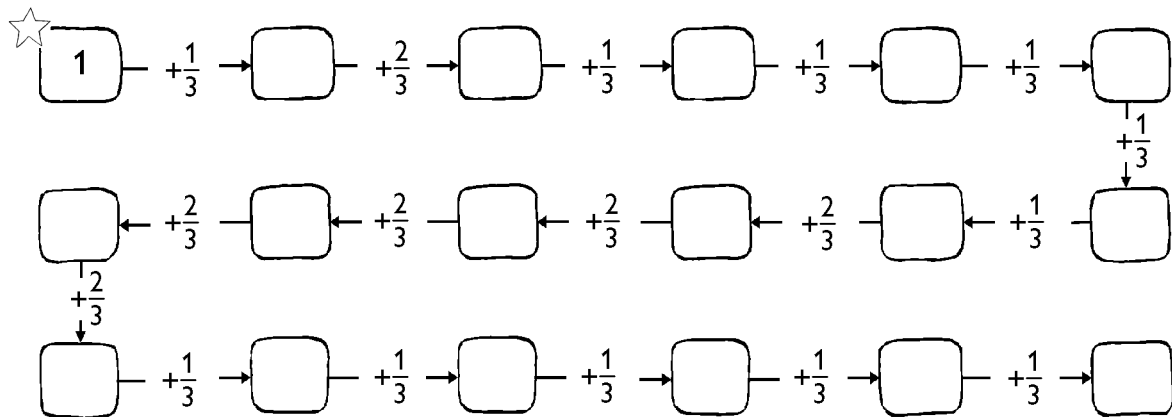


It takes $2\frac{1}{2}$ metres of material to make a dress. Fundi's mother wants to make 4 dresses. How much material does she need?

Problem 22 (Number Sense Workbook 11, page 42) – same number chain as in problem 21

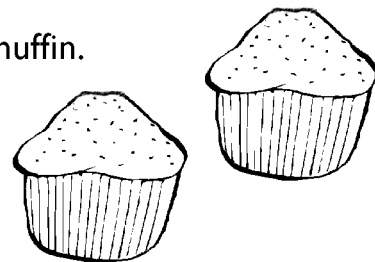
Problem 23 (Number Sense Workbook 12, page 8)

Complete.



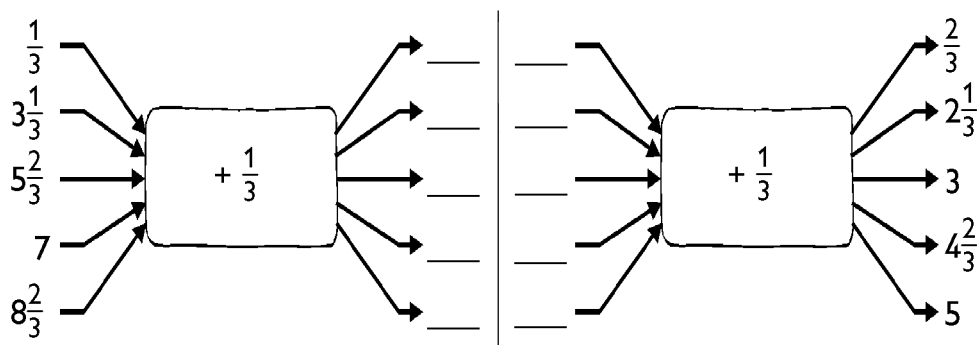
Mrs Faku needs $\frac{1}{3}$ of a cup of nuts to make one muffin.

She has 5 cups of nuts. How many muffins can she make?



Problem 24 (Number Sense Workbook 12, page 10)

Complete.



Problem 25 (Number Sense Workbook 12, page 12)

Complete the table.

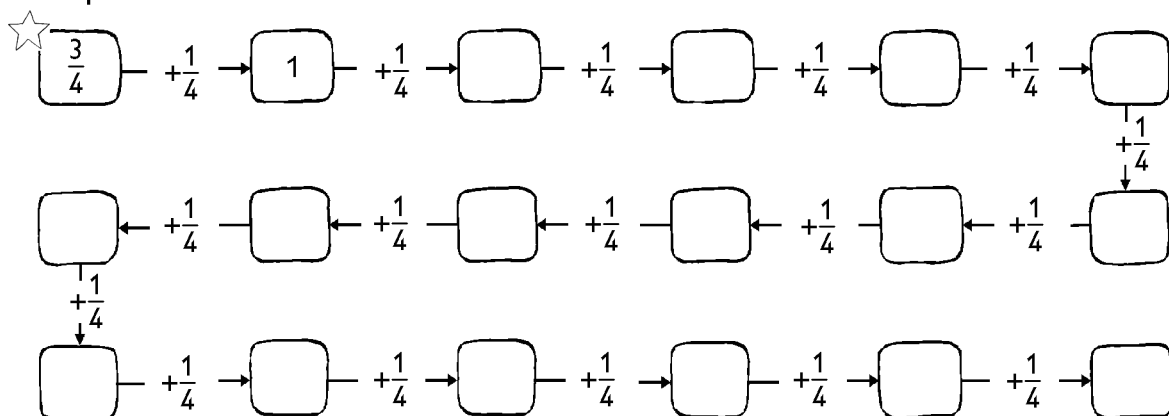
Hours	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1			3	4
Minutes				60	75	90		

Problem 26 (Number Sense Workbook 12, page 20)

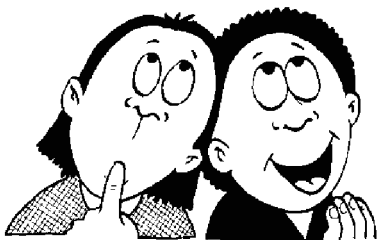
- 6 children share 8 chocolate bars equally. How much chocolate will each child get?
- 6 children share 9 chocolate bars equally. How much chocolate will each child get?

Problem 27 (Number Sense Workbook 12, page 22)

Complete.

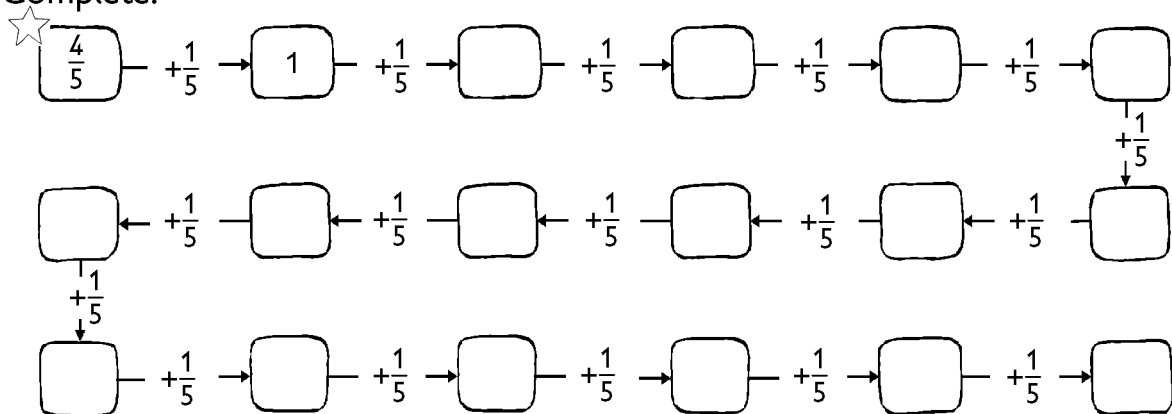


Sara needs $\frac{1}{4}$ of a metre of material to make one flag. How many flags can she make if she has 3 metres of material?

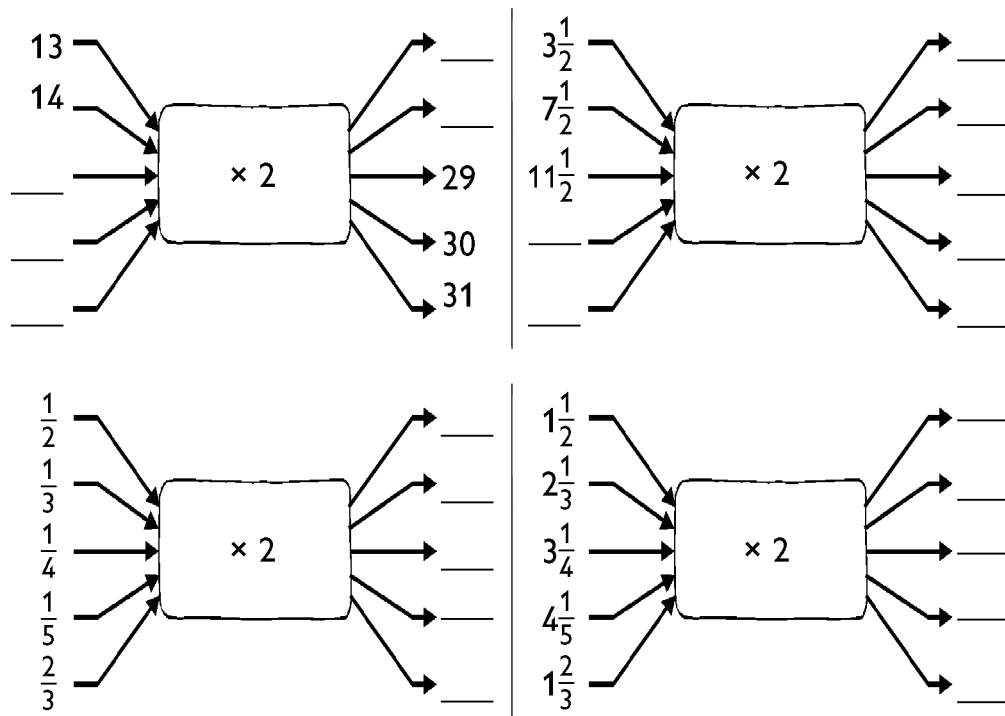


Problem 28 (Number Sense Workbook 12, page 24)

Complete.



Complete.



Problem 29 (Number Sense Workbook 12, page 31)

A steel pipe weighs $7\frac{1}{2}$ kilograms. Complete the tables.

Pipes	1	2	3	4	5	6	7	8
kg	$7\frac{1}{2}$	15						

Pipes	9	10	11	12	13	14	15	16
kg	$67\frac{1}{2}$	75						

Sequence from Grade 4 booklets

Problem 1 (Number Sense Workbook 13, page 8)

2.
 - a. Sipho and Jenny have 7 chocolate bars. They want to share the chocolate equally between them so that there is nothing left over. Show them how to do it.
 - b. Vuyo, Sam and Linda have 10 chocolate bars. They want to share the chocolate equally between them so that there is nothing left over. Show them how to do it.
 - c. Who gets more chocolate: Vuyo or Sipho?

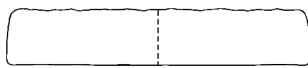

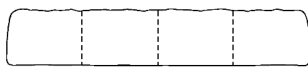

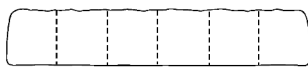
Problem 2 (Number Sense Workbook 13, page 12)

2.
 - a. Three children want to share 13 chocolate bars equally so that there is nothing left over. Show them how to do it.
 - b. Four friends want to share 13 chocolate bars equally so that there is nothing left over. Show them how to do it.
 - c. Five friends want to share 11 chocolate bars equally so that there is nothing left over. Show them how to do it.



Problem 3 (Number Sense Workbook 13, page 16)

If a chocolate bar is cut into:

- two equal parts, we call them halves 
- three equal parts, we call them thirds 
- four equal parts, we call them fourths 
- five equal parts, we call them fifths 
- six equal parts, we call them sixths 

3. a. A pizza is cut into three equal parts. What do we call each of the parts?

- b. A pizza is cut into five equal parts. What do we call each of the parts?

- c. What is more: one third of a pizza or one fifth of a pizza? _____
Why do you say that? _____

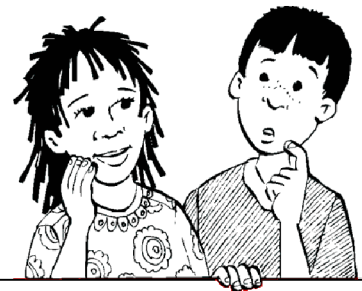
Problem 4 (Number Sense Workbook 13, page 27)

A short way of writing one half is $\frac{1}{2}$.

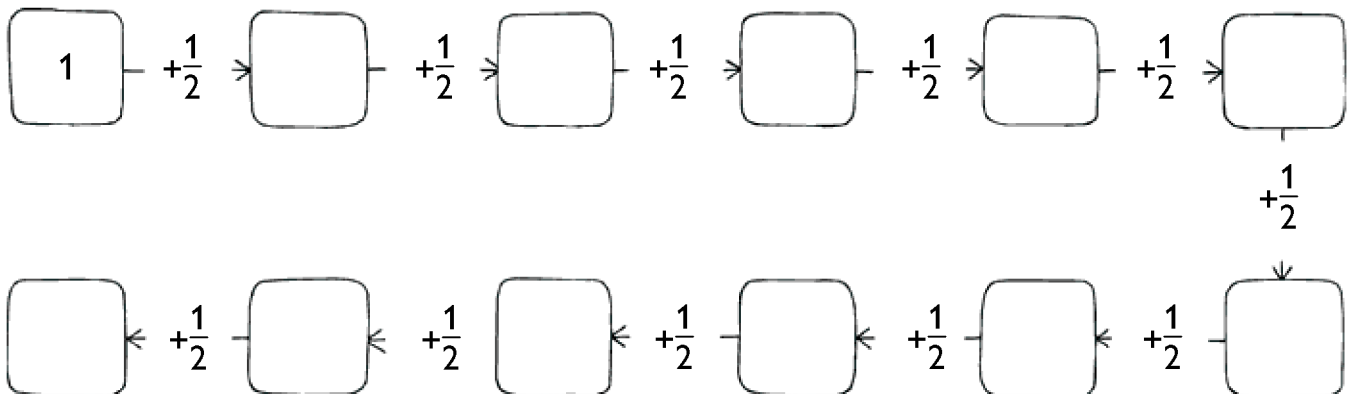
A short way of writing one third is $\frac{1}{3}$.

A short way of writing one quarter is $\frac{1}{4}$.

A short way of writing one fifth is $\frac{1}{5}$.



2. Complete.



3. 14 children are playing a netball game. The teacher wants to give a half an orange to each child at the end of the game. How many oranges must she buy?



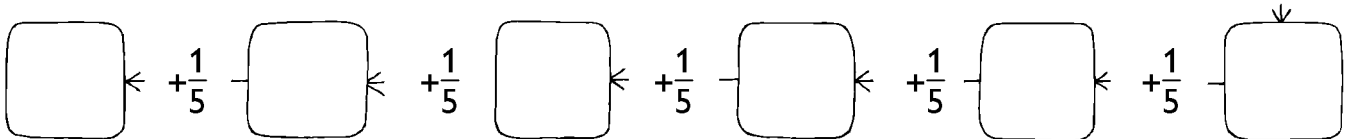
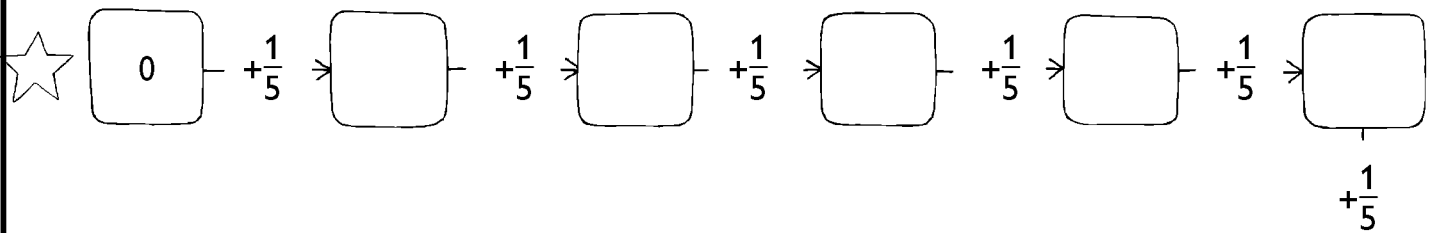
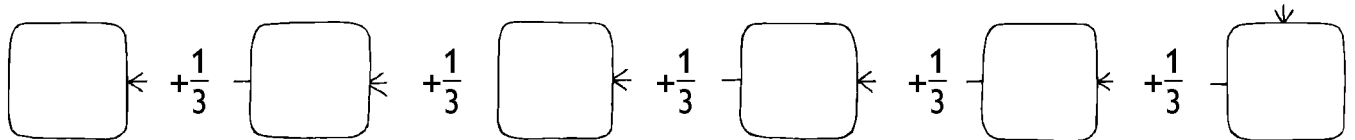
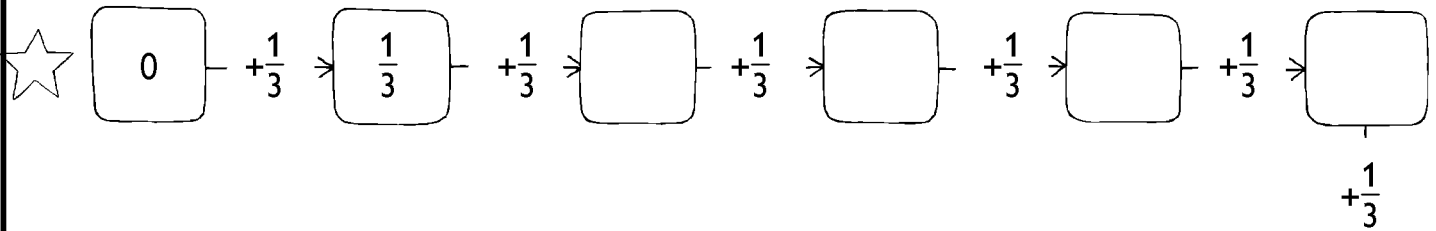
Problem 5 (Number Sense Workbook 13, page 32)

A short way of writing one third is $\frac{1}{3}$.

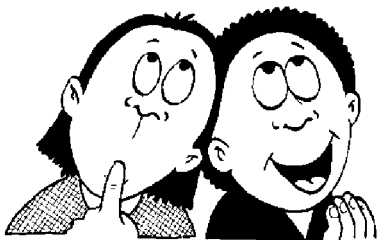
A short way of writing two thirds is $\frac{2}{3}$.



2. Complete.

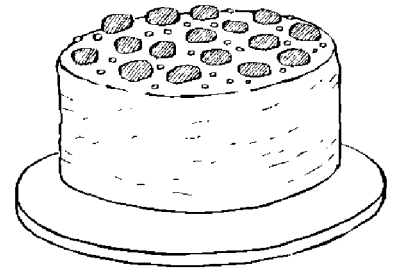


3. a. How many thirds are there in 2? ____ c. How many fifths are there in 2? ____
b. How many thirds are there in 5? ____ d. How many fifths are there in 5? ____

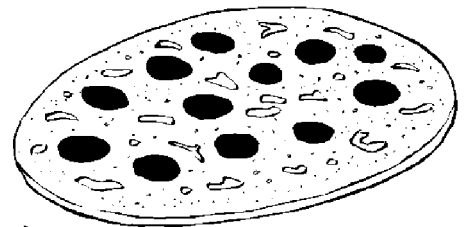


Problem 6 (Number Sense Workbook 13, page 36)

2. a. How many thirds are there in 2? _____
 b. How many thirds are there in 5? _____
 c. How many quarters are there in 2? _____
 d. How many quarters are there in 3? _____
 e. How many quarters are there in 10? _____



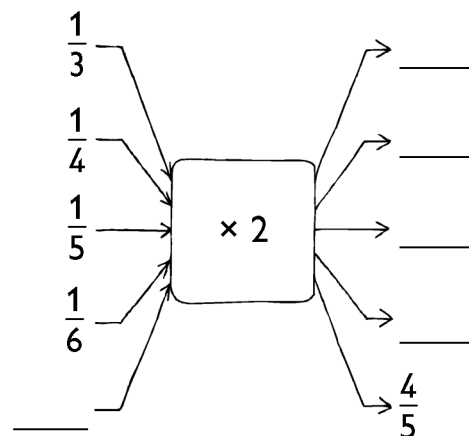
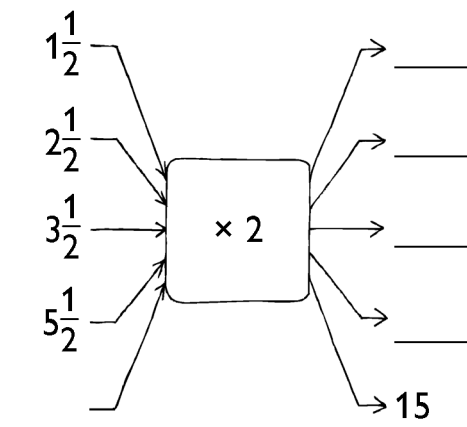
3. Mrs Faku uses $\frac{1}{3}$ of a cup nuts to make 1 cake. How many cakes can she make if she has 4 cups of nuts?

Problem 7 (Number Sense Workbook 13, page 37)

2. a. Three friends share one large pizza equally. What fraction of the large pizza does each one get?

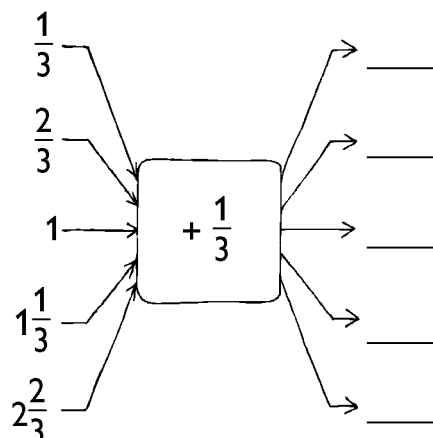
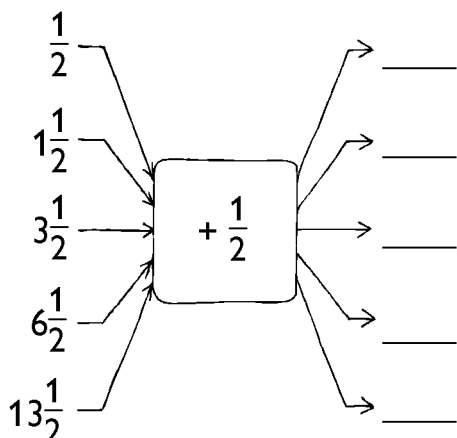
 b. Four friends share one large pizza equally. What fraction of the large pizza does each one get? _____
 c. What is bigger: one third of a pizza or one fourth (one quarter) of a pizza? ____
 Why do you say that? _____

3. Complete.



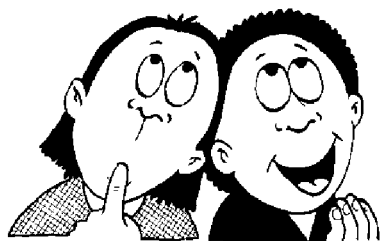
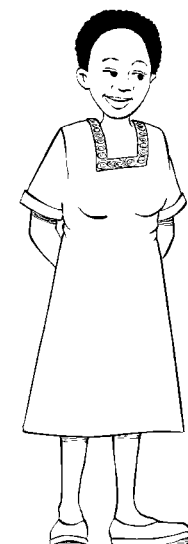
Problem 8 (Number Sense Workbook 13, page 41)

2. Complete.



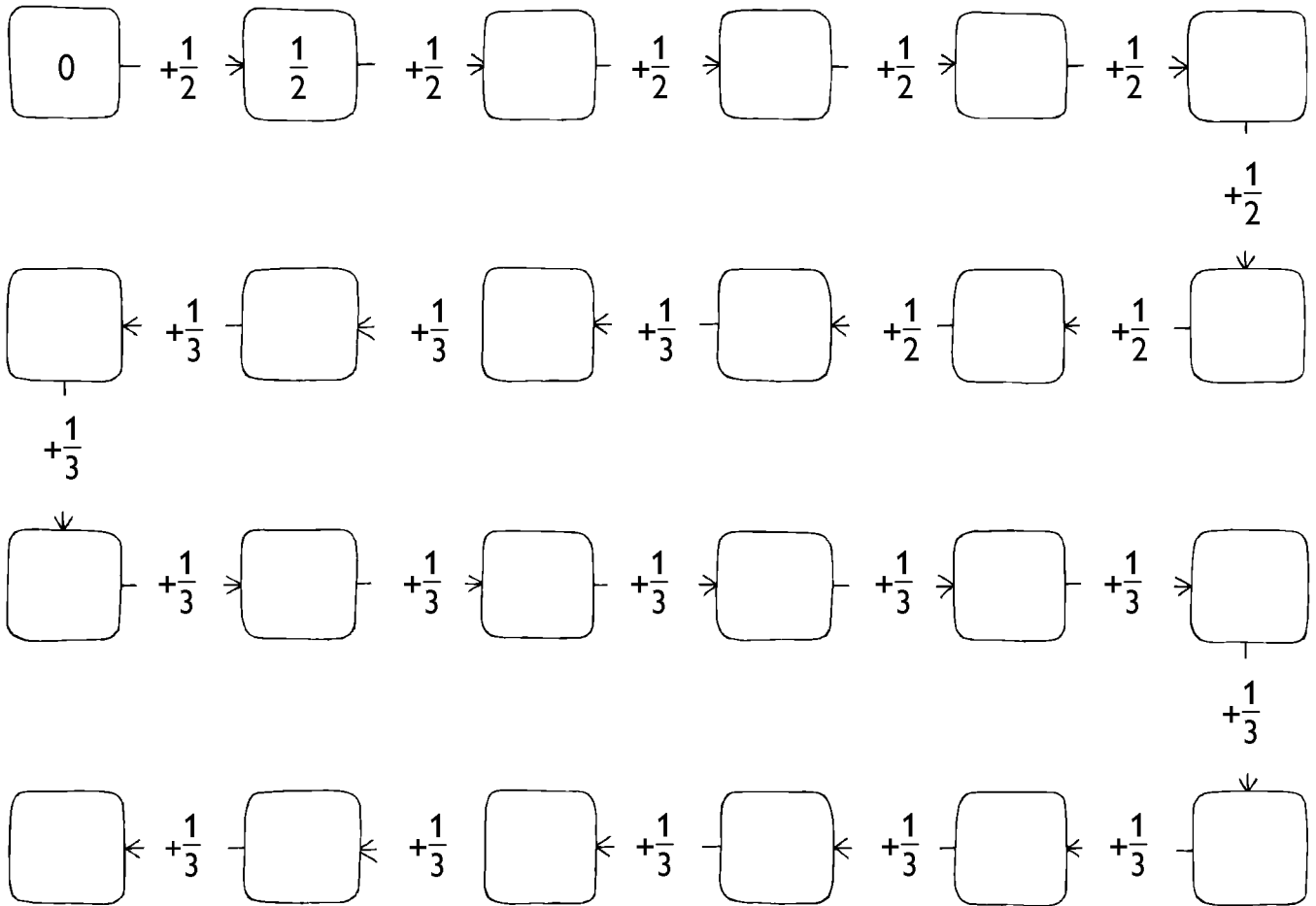
3. Mother makes porridge for breakfast. For each bowl of porridge, she uses $\frac{1}{4}$ of a litre of milk.

- If she makes 6 bowls of porridge, how many litres of milk does she need?
- Mother has 5 litres of milk. How many bowls of porridge can she prepare?

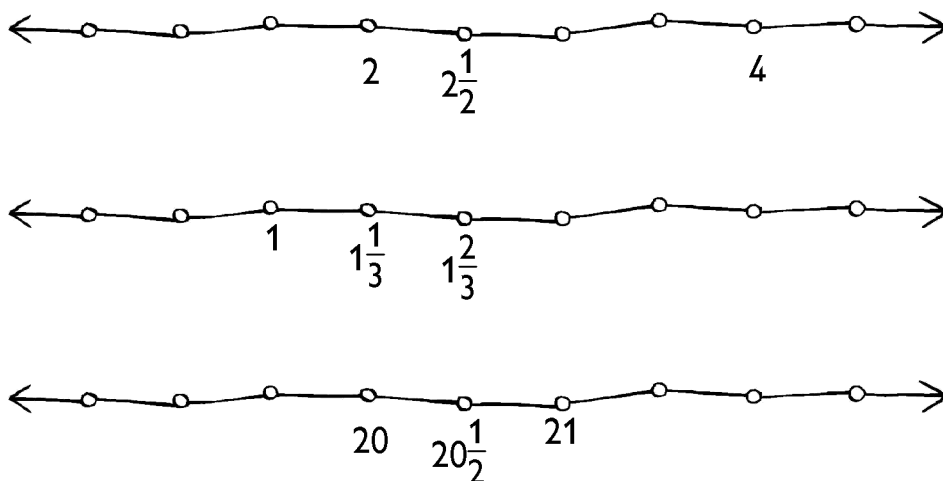


Problem 9 (Number Sense Workbook 14, page 2)

2. Complete.

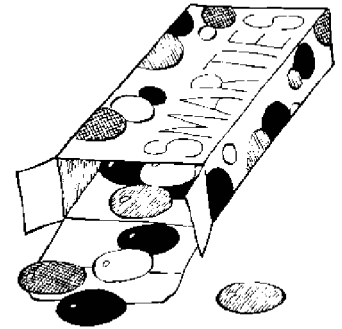


3. Complete.

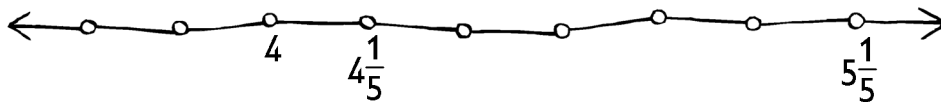


Problem 10 (Number Sense Workbook 14, page 9)

2. a. There are 24 Smarties in a box. How many Smarties are there in one-third of the box?
- b. There are 36 Smarties in a box. How many Smarties are there in one-quarter of the box?
- c. Ben and Solly share $1\frac{1}{2}$ bars of chocolate equally. How much chocolate does each boy get?

Problem 11 (Number Sense Workbook 14, page 11)

2. Complete.

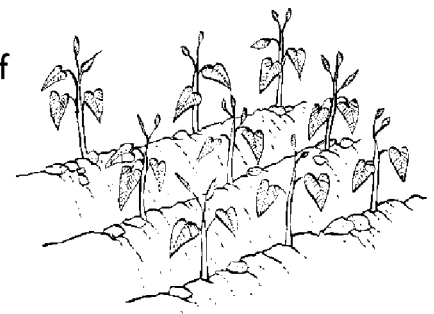


3. Vuyo uses half a loaf of bread for sandwiches every day. How many loaves does he use for sandwiches in April?
4. Three friends share half a pizza equally. Draw what each friend gets. What is each piece as a fraction of the whole pizza?

APRIL						
S	M	T	W	Th	F	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

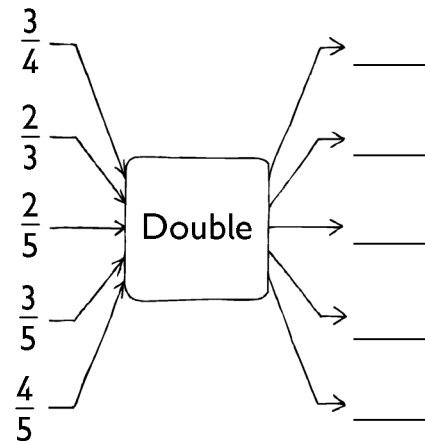
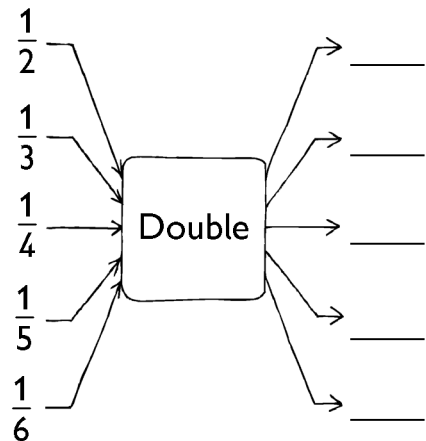
Problem 12 (Number Sense Workbook 14, page 20)

3. Mr Sibusa needs one fifth of a bag of fertilizer for one of the beds of plants in his vegetable garden.
- a. There are 10 beds in the garden. How many bags of fertilizer does he need for all his beds?
- b. He has 3 bags of fertilizer. For how many beds will that be enough?



Problem 13 (Number Sense Workbook 14, page 24)

2. Complete.

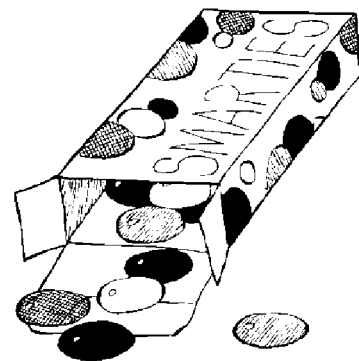


3. Lisa wants to know what is over after a party.

- How many bags of chips are left over if there are 3 big bags of chips, each $\frac{1}{2}$ full?
- How many containers of ice cream are left over if there are 2 containers of ice cream each $\frac{1}{4}$ full?
- How many jugs of juice are left over if there are 2 jugs of juice, each $\frac{2}{3}$ full?

4. There are 18 Smarties in a small box of Smarties.

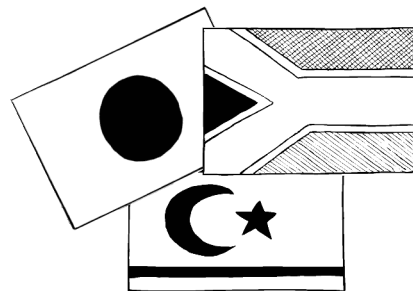
- How many Smarties is one third of a box?
- How many Smarties is two-thirds of a box?
- How many Smarties is one sixth of a box?
- How many Smarties is two-sixths of a box?
- How many Smarties is three-sixths of a box?
- Which is more: one third or two-sixths of a box of Smarties?



Why?

Problem 14 (Number Sense Workbook 14, page 28)

3. Suzi makes small flags. She needs $\frac{1}{5}$ of a metre of fabric to make one flag. She has $2\frac{1}{2}$ metres of material. How many flags can she make out of the material that she has?



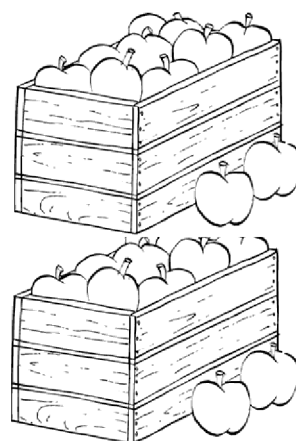
4. A rich farmer decides to donate the following to the local school:

$\frac{1}{3}$ of all his boxes of apples

$\frac{2}{3}$ of all his pockets of oranges

$\frac{1}{4}$ of all his mealies

On a certain day, the farmer has 48 boxes of apples, 75 pockets of oranges and 120 bags of mealies. How many of each of these does the school receive on this day?

Problem 15 (Number Sense Workbook 14, page 30)

2. Complete.
- ← $\frac{1}{2}$ $1\frac{3}{4}$ 3 $3\frac{1}{4}$ →
- ← $4\frac{1}{3}$ $4\frac{2}{3}$ →
- ← 3 4 →

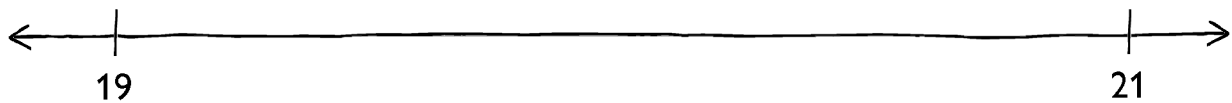
3. Approximately where would you put these numbers on this number line?

$8\frac{1}{4}$; $7\frac{5}{6}$; $8\frac{7}{8}$



4. Approximately where would you put these numbers on this number line?

$$19\frac{1}{2}; \quad 20\frac{1}{3}; \quad 20\frac{5}{6}; \quad 20\frac{1}{2}$$



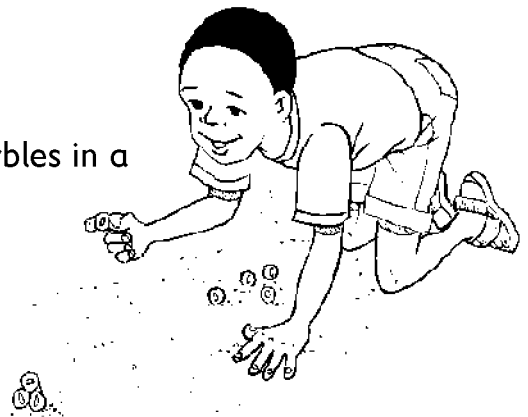
5. Ben always saves a quarter of the money he earns for looking after his sister's baby. Yesterday he saved R15. How much did he earn?

Problem 16 (Number Sense Workbook 14, page 34)

3. Colin has 20 marbles and loses a fifth of his marbles in a game.

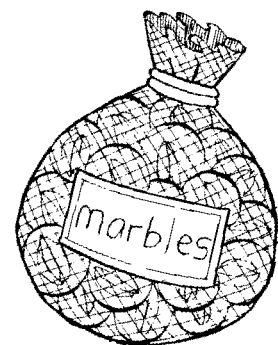
How many marbles does he lose?

Draw the 20 marbles and show a fifth of the marbles.



4. What fraction is 3 marbles of 12 marbles?

Draw the 12 marbles and show how you worked it out.

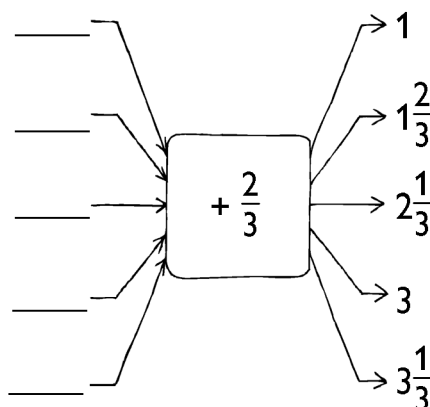
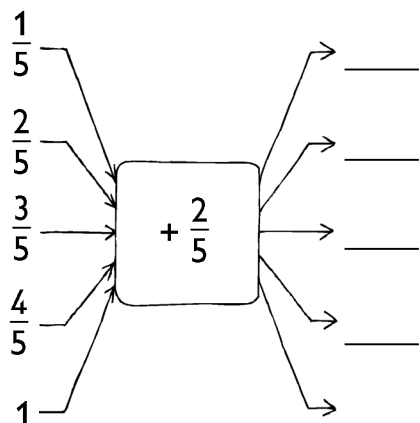


Problem 17 (Number Sense Workbook 14, page 36)

3. One fifth of a class travels to school by taxi. 7 learners in the class travel to school by taxi. How many learners are there in the class?
4. Mr Nkebe says: "One-third of my class is absent today." 15 children from Mr Nkebe's class are absent. How many children are there in his class altogether?

Problem 18 (Number Sense Workbook 14, page 39)

1. Complete.

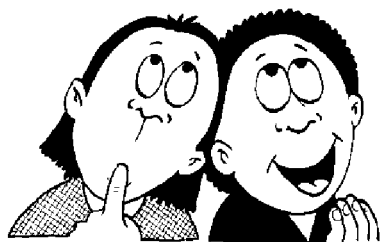
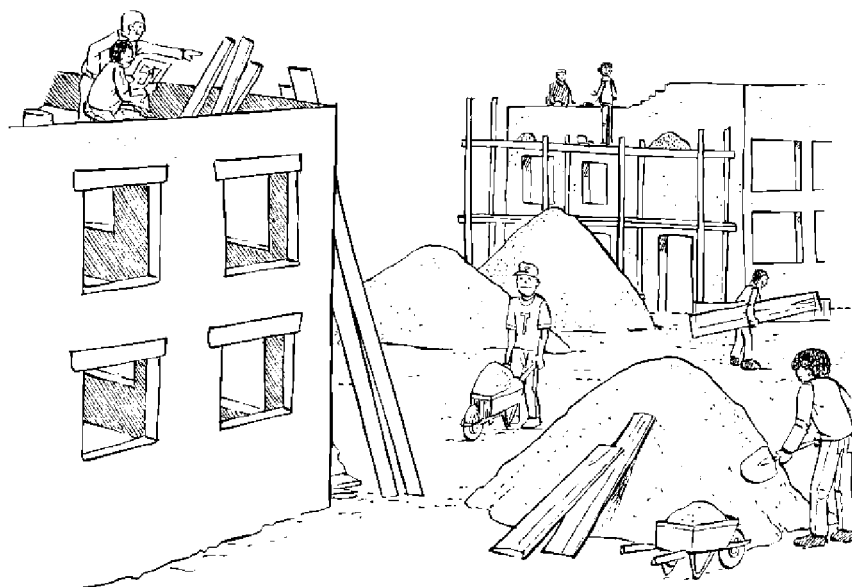


2. Write one number for each of the following.

a. $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$

b. $\frac{1}{7} + \frac{1}{7} + \frac{1}{7}$

c. $\frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9}$



Problem 19 (Number Sense Workbook 14, page 42)

1. a. Complete.

Hours	Minutes
$\frac{1}{2}$	
1	60

$\frac{1}{4}$	
$\frac{2}{4}$	
$\frac{3}{4}$	
1	

Hours	Minutes
$\frac{1}{3}$	
$\frac{2}{3}$	
1	

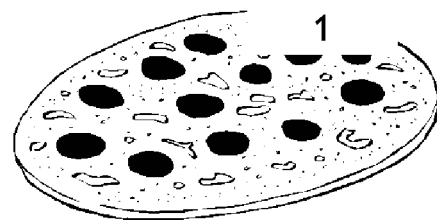


Hours	Minutes
$\frac{1}{6}$	
$\frac{2}{6}$	
$\frac{3}{6}$	
$\frac{4}{6}$	
$\frac{5}{6}$	
1	

b. Place all the fractions from the tables on the number line as carefully as you can.



2. The pizzas in the Pizza Shop are cut into eighths and quarters. Dan gets 3 quarters of a pizza. How many eighths must Vuyo get to have as much pizza as Dan?



3. Busi uses $2\frac{1}{2}$ cups of flour to bake one cake. She has 10 cups of flour. How many cakes can she bake?



Problem 20 (Number Sense Workbook 15, page 1)

1 day has 24 hours.

1 hour = $\frac{1}{24}$ of a day.



1.
 - a. Sandra does homework for 2 hours. What fraction of the day is that?
 - b. Sandra attends school for 6 hours. What fraction of the day is that?
 - c. Sandra sleeps for one-third of the day. How many hours is that?
 - d. Sandra visits her family and friends for one-eighth of the day. How many hours is that?
2. Complete.

a. $\frac{1}{3} + \frac{2}{3} = \underline{\hspace{2cm}}$	d. $\frac{5}{6} - \frac{2}{6} = \underline{\hspace{2cm}}$
b. $\frac{3}{4} + \frac{1}{2} = \underline{\hspace{2cm}}$	e. $\frac{5}{8} + \frac{3}{8} = \underline{\hspace{2cm}}$
c. $\frac{1}{5} + \frac{2}{5} = \underline{\hspace{2cm}}$	f. $\frac{4}{8} + \frac{2}{8} = \underline{\hspace{2cm}}$

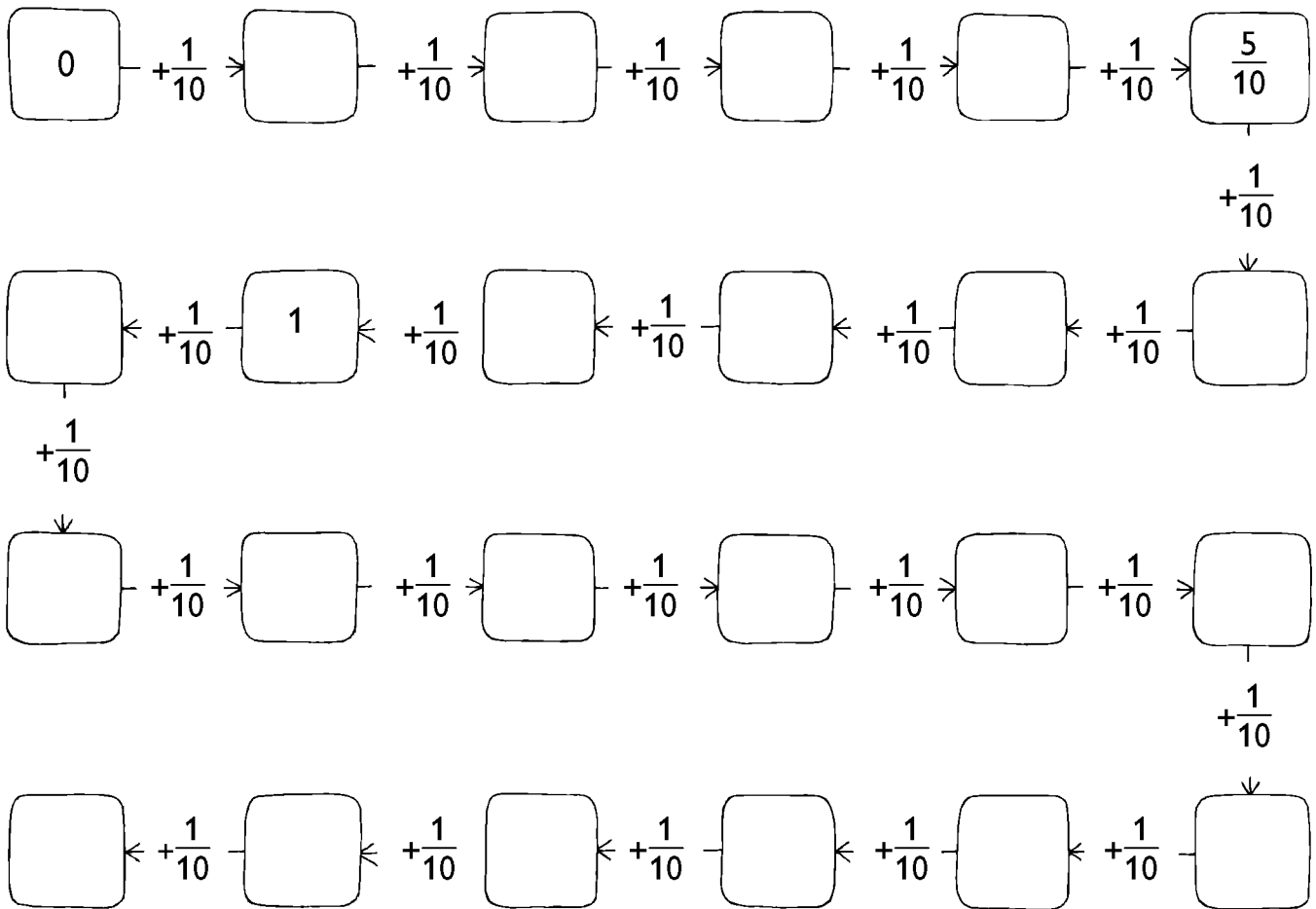
Problem 21 (Number Sense Workbook 15, page 6)

1. Complete.

a. $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} =$	b. $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} =$
c. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} =$	d. $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} =$
e. $\frac{4}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} =$	f. $\frac{5}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} =$
g. $\frac{9}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} =$	

Problem 22 (Number Sense Workbook 15, page 4)

1. Complete.



- How many tenths are there in 2? _____
- How many tenths are there in $2\frac{1}{2}$? _____
- How many tenths are there in 5? _____
- What is $\frac{1}{10} \times 5$? _____
- How many fifths are there in 2? _____

Problem 23 (Number Sense Workbook 15, page 8)

1. Complete.

$$\boxed{\frac{3}{5}} + \frac{3}{5} \Rightarrow \boxed{} + \frac{3}{5} \Rightarrow \boxed{} + \frac{3}{5} \Rightarrow \boxed{} + \frac{3}{5} \Rightarrow \boxed{3} + \frac{3}{5} \Rightarrow \boxed{} + \frac{3}{5}$$

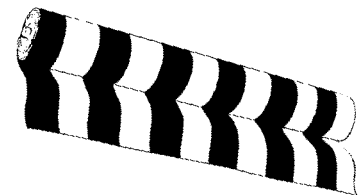
$$\boxed{7\frac{1}{5}} \leftarrow + \frac{3}{5} \leftarrow \boxed{} \leftarrow + \frac{3}{5} \leftarrow \boxed{} \leftarrow + \frac{3}{5} \leftarrow \boxed{} \leftarrow + \frac{3}{5} \leftarrow \boxed{} \leftarrow + \frac{3}{5} \leftarrow \boxed{}$$

$$\boxed{} + \frac{3}{5} \Rightarrow \boxed{} + \frac{3}{5} \Rightarrow \boxed{} + \frac{3}{5} \Rightarrow \boxed{} + \frac{3}{5} \Rightarrow \boxed{} + \frac{3}{5} \Rightarrow \boxed{} + \frac{3}{5} \Rightarrow \boxed{}$$

2. Busi uses $\frac{3}{5}$ of a metre of dress fabric to make one apron. She has 6 metres of dress fabric.

a. How many aprons can she make?

b. Complete the table.



Number of aprons	1	2	3	4	5	6	7	8	9	10
Length of fabric (m)	$\frac{3}{5}$									

Problem 24 (Number Sense Workbook 15, page 14)

2. Place these numbers on the number line as carefully as you can.

$$29\frac{1}{5} \quad 27\frac{4}{5} \quad 25\frac{1}{7} \quad 26\frac{7}{8}$$



Problem 25 (Number Sense Workbook 15, page 16)

1. A chocolate bar is cut into 3 equal pieces.



a. What fraction of the chocolate bar is each piece?

b. Each piece is cut into two equal pieces. Draw this on the diagram.



c. What fraction of the chocolate bar is each new piece?



2. Another chocolate bar is also cut into 3 equal pieces.

a. Each piece is cut into three equal pieces. Draw this on the diagram.

b. What fraction of the chocolate bar is each new piece?

3. Complete.

Number	6	12	18	24	30	36	60	120	600
$\frac{1}{6}$ of the number									

Number	6	12	18	24	30	36	60	120	600
$\frac{2}{6}$ of the number									

Number	6	12	18	24	30	36	60	120	600
$\frac{1}{3}$ of the number									

4. a. Mr Fourie promises to give one tenth of his peaches to the school. He has 90 trays of peaches. How many trays must he give the school?
- b. Mr Adams gives $\frac{1}{3}$ of a box of chips to Vuyo's class. If there are 90 packets of chips in the box, how many packets of chips does the class get?
- c. Aunt Sally makes small apple tarts. She uses $\frac{3}{4}$ of an apple for one tart. How many tarts can she make if she has 10 apples available?

Problem 26 (Number Sense Workbook 15, page 18)

1. Complete.

$$\boxed{2\frac{1}{5}} \xrightarrow{+\frac{1}{5}} \boxed{} \xrightarrow{+\frac{2}{5}} \boxed{} \xrightarrow{} \boxed{3} \xrightarrow{} \boxed{3\frac{1}{3}} \xrightarrow{} \boxed{3\frac{2}{3}}$$

$$\boxed{5\frac{1}{2}} \xleftarrow{+\frac{1}{4}} \boxed{5\frac{1}{4}} \xleftarrow{+\frac{1}{4}} \boxed{} \xleftarrow{+\frac{1}{4}} \boxed{} \xleftarrow{+\frac{1}{4}} \boxed{4\frac{1}{2}} \xleftarrow{} \boxed{4}$$

$$\boxed{} \xrightarrow{} \boxed{6} \xrightarrow{} \boxed{6\frac{1}{6}} \xrightarrow{} \boxed{6\frac{5}{6}} \xrightarrow{} \boxed{7} \xrightarrow{+\frac{1}{2}} \boxed{} \xrightarrow{+\frac{1}{2}} \boxed{}$$

$$\boxed{11} \xleftarrow{+\frac{5}{8}} \boxed{} \xleftarrow{+\frac{3}{8}} \boxed{} \xleftarrow{+\frac{3}{8}} \boxed{9} \xleftarrow{+\frac{1}{3}} \boxed{} \xleftarrow{+\frac{1}{3}} \boxed{}$$

2. Complete.

a. $\frac{3}{5} + \frac{1}{5} = \underline{\hspace{2cm}}$ d. $2 - \frac{1}{5} = \underline{\hspace{2cm}}$ g. $4\frac{1}{7} - \frac{2}{7} = \underline{\hspace{2cm}}$

b. $1\frac{4}{5} + \frac{1}{5} = \underline{\hspace{2cm}}$ e. $2\frac{1}{5} - \frac{2}{5} = \underline{\hspace{2cm}}$ h. $8\frac{1}{3} - 1\frac{1}{3} = \underline{\hspace{2cm}}$

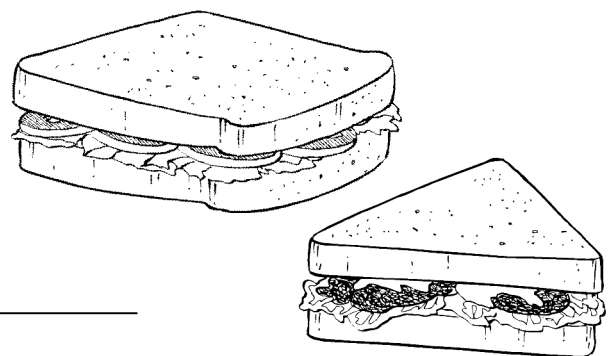
c. $1\frac{2}{7} + 1\frac{5}{7} = \underline{\hspace{2cm}}$ f. $8\frac{1}{3} - \underline{\hspace{2cm}} = 7\frac{2}{3}$

3. Maria uses $\frac{2}{3}$ of a loaf of bread every day to make sandwiches for her family.

a. How much bread will she use in 5 days?

b. How many loaves of bread must she buy?

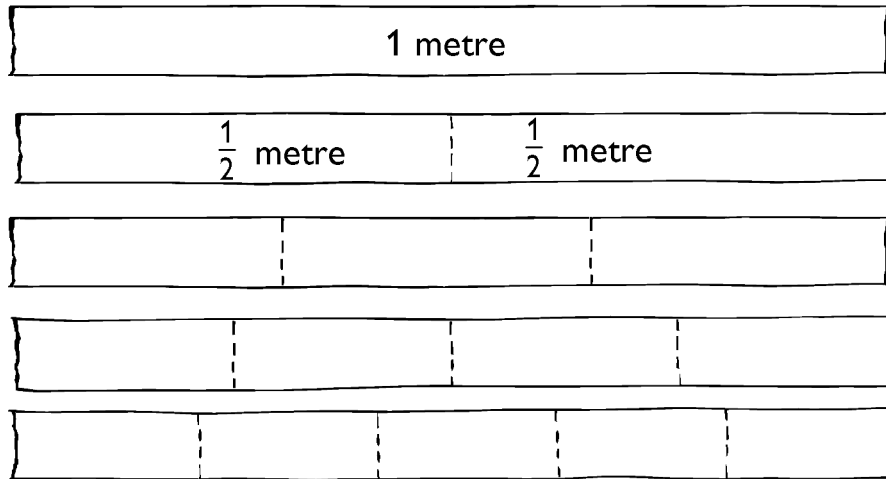
c. Maria has 2 loaves of bread. For how many days can she make sandwiches? Use a drawing to show how you determined your answer.



Problem 27 (Number Sense Workbook 15, page 24)

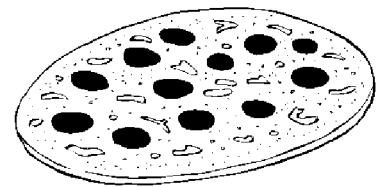
2. 1 metre strips of ribbon are cut into different sized pieces.

Name each piece. Write the names on the diagram.

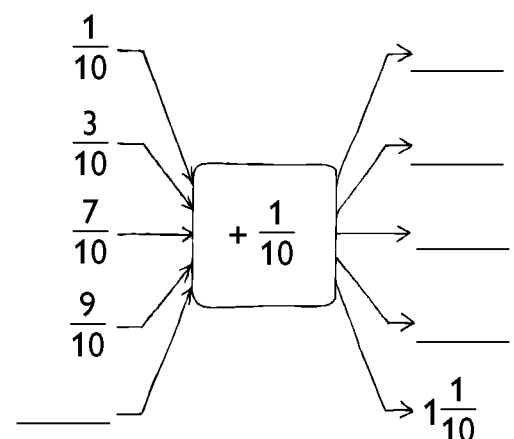
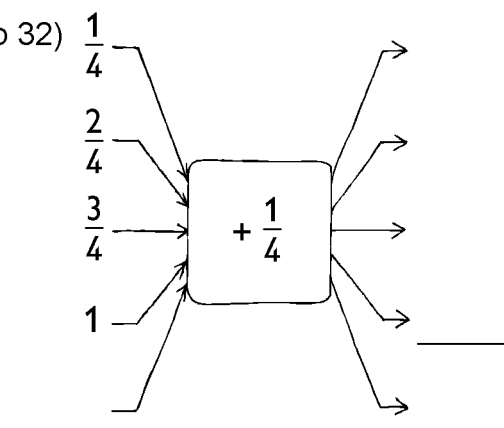


Which piece of ribbon is the longest?

- a. $\frac{1}{2}$ metre or $\frac{1}{4}$ metre _____ c. $\frac{1}{3}$ metre or $\frac{1}{5}$ metre _____
- b. $\frac{1}{2}$ metre or $\frac{2}{4}$ metre _____ d. $\frac{1}{2}$ metre or $\frac{3}{5}$ metre _____
3. In a certain competition, everybody pays an entry fee. The first prize is one-quarter of the money raised by the entry fees. The second prize is half of the first prize. What fraction of the entry fee is the second prize?
4. Dan buys one-third of a pizza. Vuyo wants one half of Dan's pizza. What fraction of the whole pizza does Vuyo get?

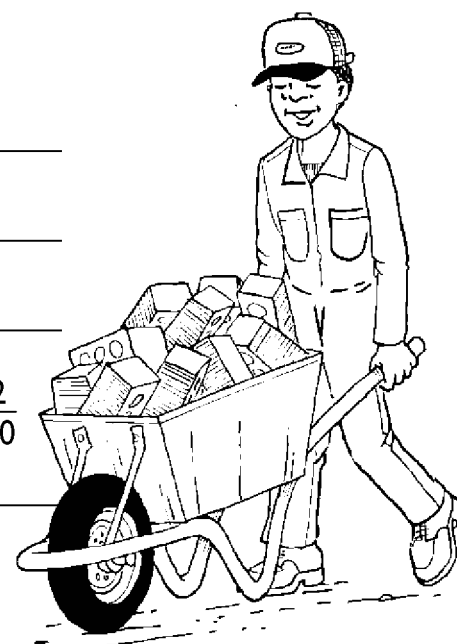
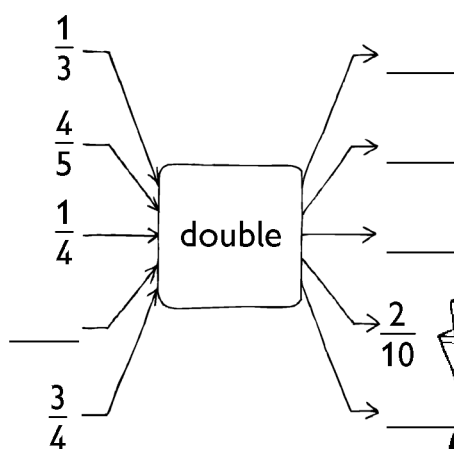
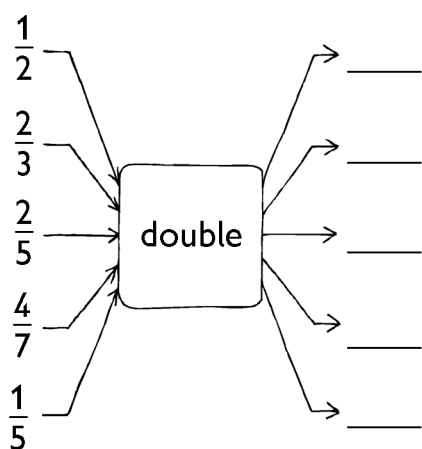
Problem 28 (NSW15, p 32)

2. Complete.



Problem 29 (Number Sense Workbook 15, page 43)

1. Complete.

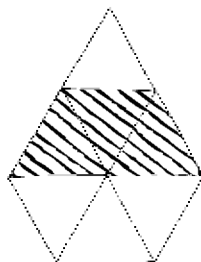


Problem 30 (Number Sense Workbook 15, page 44)

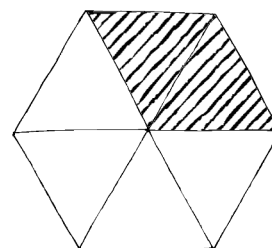
1. In each of the following cases write down what fraction of the figure is shaded.



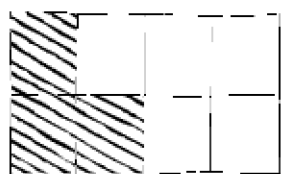
a. $\frac{2}{5}$



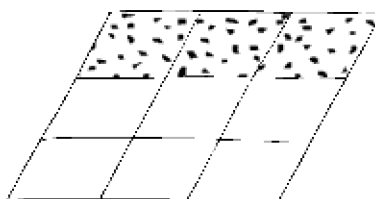
b. _____



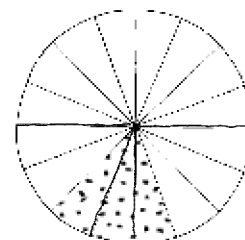
c. _____



d. _____



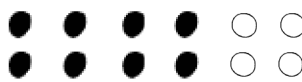
e. _____



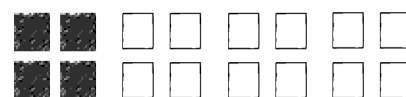
f. _____



g. _____



h. _____



i. _____

2. Complete.

a. $\frac{3}{8} + \frac{5}{8} = \underline{\hspace{2cm}}$

b. $\frac{4}{5} + \frac{1}{5} = \underline{\hspace{2cm}}$

$\frac{3}{8} + \frac{6}{8} = \underline{\hspace{2cm}}$

$\frac{4}{5} + \frac{2}{5} = \underline{\hspace{2cm}}$

$\frac{3}{8} + \frac{7}{8} = \underline{\hspace{2cm}}$

$\frac{4}{5} + \frac{4}{5} = \underline{\hspace{2cm}}$



Conversions

1m = 100 cm

1 hour = 60 minutes

3. Complete.

a. $\frac{1}{5}$ of a metre = $\underline{\hspace{2cm}}$ cm

f. $1\frac{1}{2}$ metres = $\underline{\hspace{2cm}}$ cm

b. $\frac{2}{5}$ of a metre = $\underline{\hspace{2cm}}$ cm

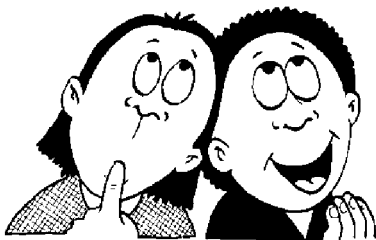
g. $\frac{1}{5}$ of an hour = $\underline{\hspace{2cm}}$ minutes

c. $\frac{1}{4}$ of a metre = $\underline{\hspace{2cm}}$ cm

h. $\frac{3}{4}$ of an hour = $\underline{\hspace{2cm}}$ minutes

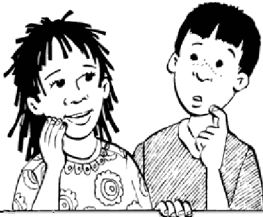
d. $\frac{3}{4}$ of a metre = $\underline{\hspace{2cm}}$ cm

e. $\frac{1}{10}$ of a metre = $\underline{\hspace{2cm}}$ cm



Problem 31 (Number Sense Workbook 15, page 46)

2. a. Complete the tables.



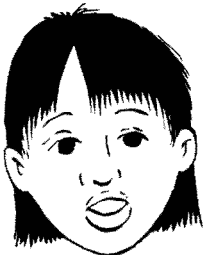
Conversions
1 day = 24 hours

Days	Hours
$\frac{1}{2}$	
1	24

Days	Hours
$\frac{1}{4}$	
$\frac{2}{4}$	
$\frac{3}{4}$	
1	

Days	Hours
$\frac{1}{8}$	
$\frac{2}{8}$	6
$\frac{3}{8}$	
$\frac{4}{8}$	
$\frac{5}{8}$	
$\frac{6}{8}$	
$\frac{7}{8}$	
1	

To write $\frac{1}{4} + \frac{1}{2}$ as one fraction I see
that $\frac{1}{2} = 12$ hours and $\frac{2}{4} = 12$ hours
so $\frac{1}{4} + \frac{1}{2} = \frac{1}{4} + \frac{2}{4} = \frac{3}{4}$



- b. Write as one fraction.

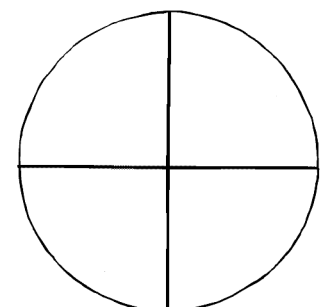
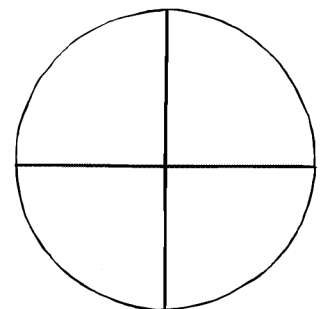
$$\frac{1}{4} + \frac{1}{8} = \frac{1}{2} + \frac{3}{8} =$$

$$\frac{1}{4} + \frac{3}{8} = \frac{1}{2} + \frac{4}{8} =$$

$$\frac{3}{4} + \frac{1}{8} = \frac{3}{4} + \frac{1}{8} =$$

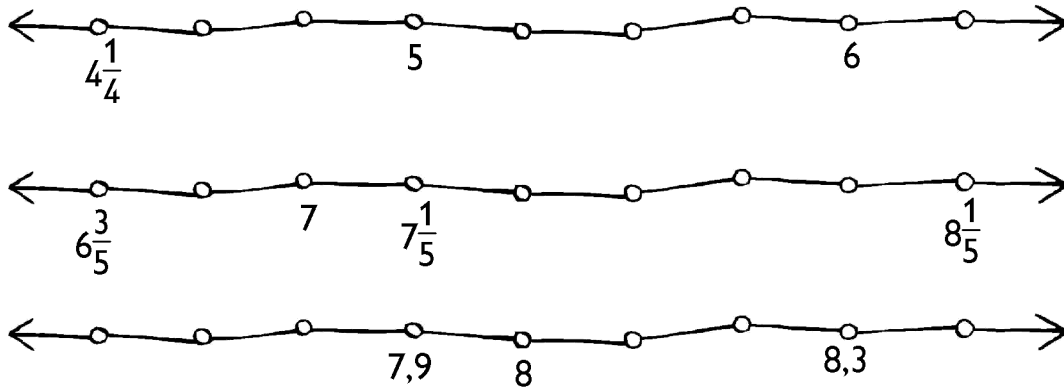
Problem 32 (Number Sense Workbook 16, page 4)

2. A pizza is cut into 4 equal pieces.
- What fraction of the pizza is each of the pieces? _____
 - Each of the pieces is again cut into 2 equal pieces. What fraction of the pizza is each of the new pieces?
 - Show the pieces on the diagram alongside.
 - Another pizza is cut into 4 equal pieces. Each piece is cut into 3 equal pieces. What fraction of the pizza is each of the new pieces?
 - Show the pieces on the diagram alongside.



Problem 33 (Number Sense Workbook 16, page 5)

3. Complete.

Problem 34 (Number Sense Workbook 16, page 8)

3. Complete.

- a. $\frac{1}{6}$ of an hour = _____ b. $\frac{1}{3}$ of an hour = _____ d. $\frac{1}{6} + \frac{1}{6}$ of an hour = _____
- $\frac{2}{6}$ of an hour = _____ $\frac{2}{3}$ of an hour = _____ $\frac{1}{3} + \frac{1}{6}$ of an hour = _____
- $\frac{3}{6}$ of an hour = _____ c. $\frac{1}{4}$ of an hour = _____
- $\frac{4}{6}$ of an hour = _____ $\frac{2}{4}$ of an hour = _____
- $\frac{5}{6}$ of an hour = _____ $\frac{3}{4}$ of an hour = _____

Problem 35 (Number Sense Workbook 16, page 13)

2. One cup is the same as 250ml.
 One teaspoon is the same as 5ml.
 One tablespoon is the same as 15ml.



Write all the quantities in this recipe in millilitres.

SPICY BISCUITS

- 4 cups flour _____ $\frac{2}{5}$ teaspoon salt _____
 1 cup sugar _____ $\frac{1}{4}$ tablespoon cream of tartar _____
 $\frac{1}{4}$ cup butter _____ $\frac{1}{2}$ teaspoon ground cloves _____
 $\frac{1}{4}$ cup soft fat _____ 1 teaspoon ground cinnamon _____
 1 egg $\frac{2}{5}$ cup water _____

Problem 36 (Number Sense Workbook 16, page 15)

2. Complete.

Number	6	12	18	24	30	60	72	90	300	600
$\frac{1}{6}$ of the number										
$\frac{2}{6}$ of the number										
$\frac{1}{3}$ of the number										

Problem 37 (Number Sense Workbook 16, page 17)

2. Complete.

a. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} =$

b. $\frac{2}{3} + \frac{2}{3} =$

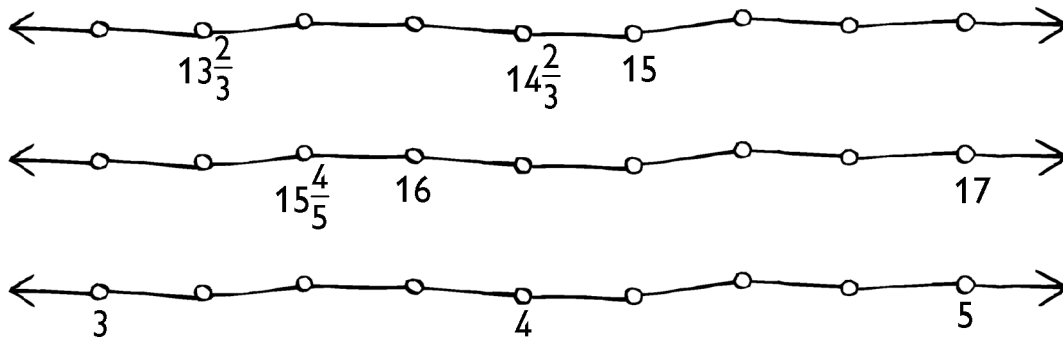
c. $\frac{5}{6} + \frac{1}{6} + \frac{1}{6} =$

d. $1\frac{3}{8} - \frac{4}{8} =$

e. $1\frac{1}{3} - \frac{2}{3} =$



3. Complete,

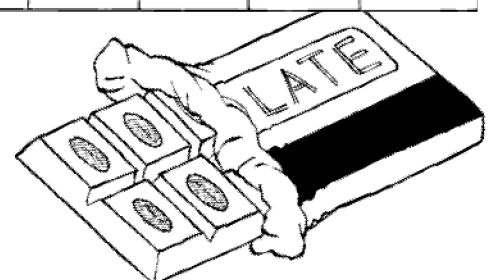
Problem 38 (Number Sense Workbook 16, page 34)

2. A slab of chocolate is made up of 12 blocks.

- a. How many people can easily share the chocolate slab equally among them?
Use the table to organise your thinking as you investigate all possible solutions.

Number of people						
No. of blocks per person						
Fraction of the chocolate slab per person						

- b. Did you find all the possibilities?
c. How do you know that you have found all the possible solutions?
d. Can you write the fractions in the last row in more than one way?



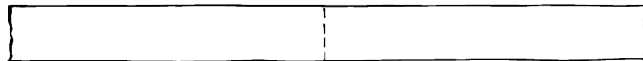
- e. The chocolate slab is shared equally among three people. Lucy says each person gets $\frac{1}{3}$ of the slab. Joe says each person gets $\frac{4}{12}$ of the slab. Who is correct and who is incorrect? Explain.

Problem 39 (Number Sense Workbook 16, page 39)

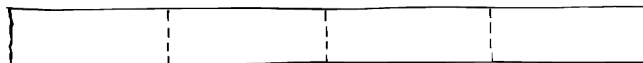
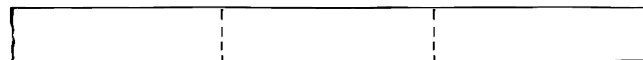
2. Here are ribbons that have been cut.

Use the ribbons to help you decide which fraction is bigger.

a. $\frac{1}{3}$ or $\frac{2}{6}$



b. $\frac{1}{7}$ or $\frac{1}{8}$



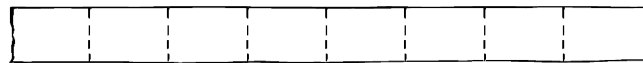
c. $\frac{1}{2}$ or $\frac{3}{5}$



d. $\frac{7}{15}$ or $\frac{4}{8}$



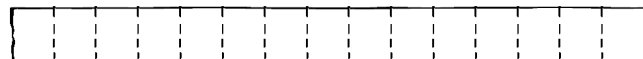
e. $\frac{2}{4}$ or $\frac{3}{6}$



f. $\frac{9}{18}$ or $\frac{6}{10}$



g. $\frac{5}{10}$ or $\frac{2}{5}$



Problem 40 (Number Sense Workbook 16, page 46)

2. Themba and Xolile buy ordinary candles at the supermarket. They melt the candles and make fancy candles from the wax. They make two different fancy candles.

A small round candle is made from $\frac{2}{3}$ of an ordinary candle.

A small square candle is made from $\frac{3}{4}$ of an ordinary candle.

- a. How many ordinary candles do they need to buy to make 10 small round candles?
- b. How many ordinary candles do they need to buy to make 10 small square candles?

Selected other problemsProblem 42 (Number Sense Workbook 17, page 4)

- 2.
- a. Six children share 19 vienna sausages equally so that there is nothing left over. How much sausage will each child get?
 - b. Six children share 20 vienna sausages equally so that there is nothing left over. How much sausage will each child get?



Problem 43 (Number Sense Workbook 17, page 14)

1. Complete.

$$\boxed{0} + \frac{2}{3} \Rightarrow \boxed{\frac{2}{3}} + \frac{2}{3} \Rightarrow \boxed{} + \frac{2}{3} \Rightarrow \boxed{} + \frac{2}{3} \Rightarrow \boxed{\frac{2}{3}} + \frac{2}{3} \Rightarrow \boxed{} + \frac{2}{3}$$

$$\boxed{} + \frac{2}{3} \Rightarrow \boxed{} + \frac{2}{3} \Rightarrow \boxed{6} + \frac{2}{3} \Rightarrow \boxed{} + \frac{2}{3} \Rightarrow \boxed{} + \frac{2}{3} \Rightarrow \boxed{} + \frac{2}{3}$$

$$\boxed{} + \frac{2}{3} \Rightarrow \boxed{} + \frac{2}{3} \Rightarrow \boxed{} + \frac{2}{3} \Rightarrow \boxed{} + \frac{2}{3} \Rightarrow \boxed{} + \frac{2}{3} \Rightarrow \boxed{} + \frac{2}{3}$$

2. a. How many two-thirds are there in 6?

b. How many thirds are there in 6?

 c. What is $9 \times \frac{2}{3}$?

3. a. A car travels at a constant speed of 80 km per hour. Complete the table.

Time (hours)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	2	3	4	5	6
Distance travelled (km)					160				

b. Now complete the table for a car that travels at 120 km per hour.

Time (hours)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	2	3	4	5	6
Distance travelled (km)					240				

 4. Mrs Manga uses $\frac{2}{3}$ of a cup of nuts to bake 1 cake.

a. How many cups of nuts did she use if she made 20 cakes?

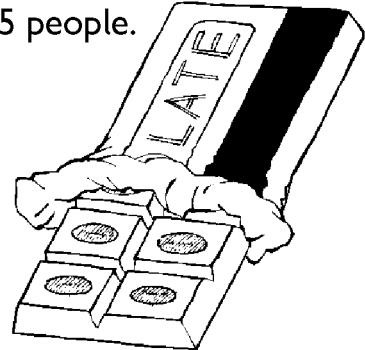
b. How many cakes can she bake if she has 10 cups of nuts?

Problem 44 (Number Sense Workbook 19, page 8)

Think about different chocolate bars and chocolate slabs. Some of them are designed so that it is easy to break off one or more pieces.



1. Design 2 chocolate slabs that can easily be shared (equally) by 5 people.

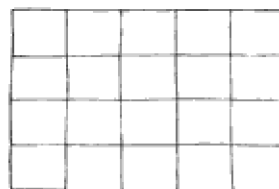


Puleng and Tom have each designed a chocolate slab that can easily be shared (equally) by 5 people.

Puleng's slab:

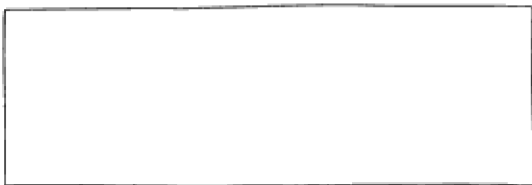


Tom's slab:

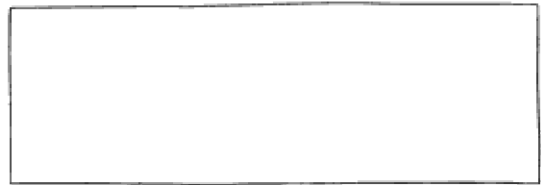


2. Use the rectangles to design two more slabs that can easily be shared (equally) by 5 people. Each slab should have a different number of blocks.

A.



B.



3. What fraction of the whole slab is each small block in:
- Puleng's slab?
 - Tom's slab?
 - Slab A?
 - Slab B?
4. Design a chocolate slab so that five people each get 3 blocks if the slab is shared equally among them. Name the fraction of the slab that each person gets in two different ways.
5. Design a slab that can be shared equally among 5 people and 7 people. Describe how you did it.

Problem 45 (Number Sense Workbook 19, page 12)

2. Pizza slices are sold in three sizes: Maxi, Midi and Mini.


A large pizza is cut in five equal pieces to make 5 Maxi slices.

A large pizza is cut in ten equal pieces to make 10 Midi slices.

A large pizza is cut in fifteen equal pieces to make 15 Mini slices.

- Joey buys two Maxi slices.
 - Jack buys four Midi slices.
 - Jane buys six Mini slices.
 - Donald buys one Maxi slice and one Midi slice.
- a. Who bought the most pizza? Explain.
 - b. What fraction of a large pizza did Donald buy altogether?
 - c. Did Jane buy more or less pizza than Donald did?
 - d. Is it possible to combine Maxi, Midi and Mini slices to make up one third of a large pizza? Explain.

Problem 46 (Number Sense Workbook 19, page 28)

1. A slab of chocolate is made up of 24 blocks.
 - a. What fraction of the slab of chocolate is one small block?
 - b. How many people can easily share the chocolate slab equally among them?
Use the table to organise your thinking as you investigate all possible solutions. If you can, write the fraction of the slab in more than one way.
 - c. Did you find all the possibilities?
 - d. The slab is shared equally among four people. Diane says each person gets $\frac{1}{4}$ of the slab. Joe says each person gets $\frac{6}{24}$ of the slab. Who is correct and who is incorrect? Explain.
2. Lebo spends $\frac{1}{3}$ of his day sleeping and $\frac{1}{4}$ of his day at school.
 - a. Does he spend more time sleeping or more time at school? Explain.

 - b. How much time does he have left over for other activities?
3. Use your working for question 1 to help you determine:

Is there one fraction of a slab that is the same as $\frac{1}{3}$ of a slab plus $\frac{1}{4}$ of a slab?
Explain your answer.

Analysing fraction problems

We analyse fraction problems in terms of the concepts/skills/big ideas that they are developing. These include:

- Equal sharing
- Cutting a whole into any number of parts
- Fraction names
- Determining the name of a part
- Comparing relative size
- Non-unitary fractions
- Fraction notation
- Combining parts
- Counting in fractions
- Reasoning about grouping
- Fraction of a collection
- Fraction of a number
- Repeated addition contexts
- Repeated addition number contexts
- Existence of equivalence
- Completing a number line
- Position on a (empty) number line
- Inverse fraction of a collection
- Fraction of a fraction
- Operations with fractions
 - Equivalence
 - Addition and subtraction
 - Multiplication
 - Division

