Making the most of the Number Sense Workbooks

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1. Bongi makes 3-legged stools and 4-legged chairs. Last month he used 72 legs altogether. He made 3 more stools than chairs. How many stools and how many chairs did he make? Investigate.

Workbook 16 page 40

Develop three variations of the problem:

- One that is easier – for children who are struggling to get started
- One that is similar to test for understanding
- One that is harder – to test for understanding and for children who need to be extended

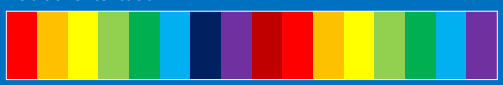
The Number Sense Workbook Series (assumptions and philosophy)

- The Number Sense Workbooks consist of 192 worksheets per year that provide support for:
  - Differentiated teaching,
  - Independent learning, and
  - Experiencing mathematics as a sense-making activity.

The Number Sense Workbooks

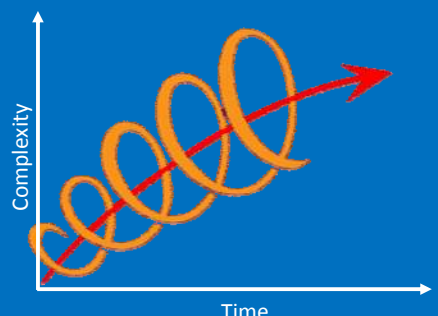
- The Number Sense Workbooks:
  - Cover the curriculum for:-
    - Number,
    - Patterns, and
    - Measurement (for grades 4 to 7)
- The Number Sense Workbooks do not:
  - Cover the curriculum for: Space and Shape & Data Handling
  - They do not arrange the materials by curriculum topic

- Traditional textbook:



- Number Sense Workbook:

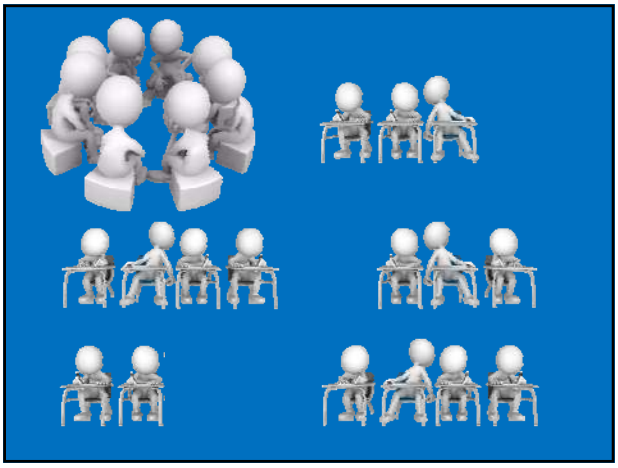
  - Mathematical structure, especially in the early grades, is constellatory (interrelated/integrated) in nature
  - Children learn by regular visiting and re-visiting (interiorisation, condensation and reification)



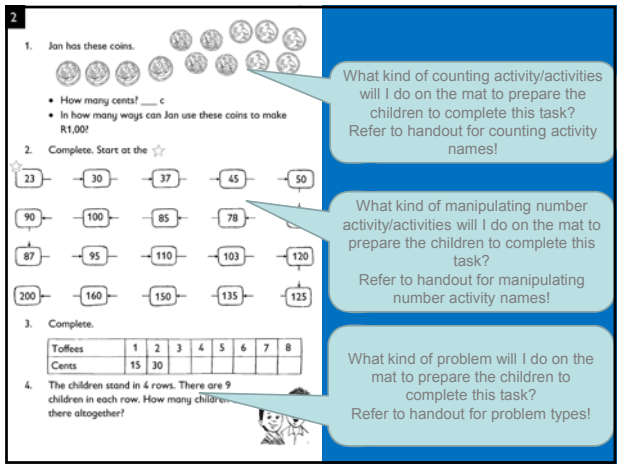
The Number Sense Workbook Series implications for teaching:

- Differentiated teaching:
  - Classroom organisation
- Independent learning:
  - The role of the child
- Experiencing mathematics as a sense-making activity
  - The nature of the mathematics

Differentiated teaching – classroom organisation



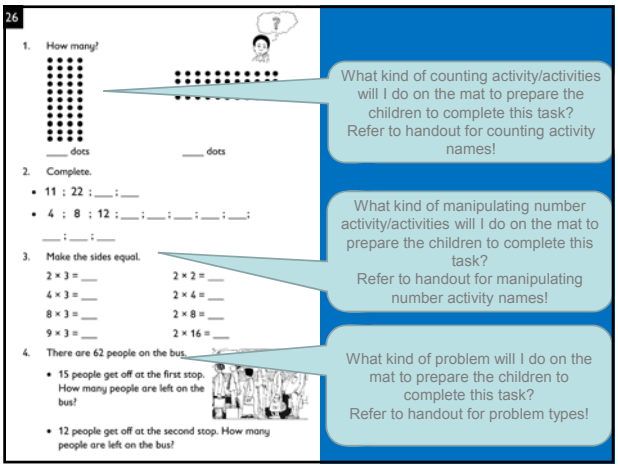
	Session 1 (20 mins)	Session 2 (20 mins)	Session 3 (20 mins)
Grp 1	Mat (correct p1 anticipate p2)	•Correct p1 •Start p2	•Continue p2
Grp2	•Complete p25	Mat (discuss p25 anticipate p26)	•Correct p25 •Start p26
Grp 3	•Correct p30 •Start p31	•Continue p31	Mat (discuss p31 anticipate p32)

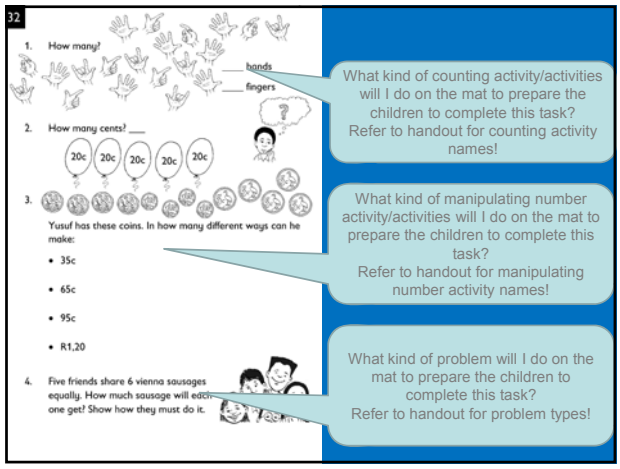


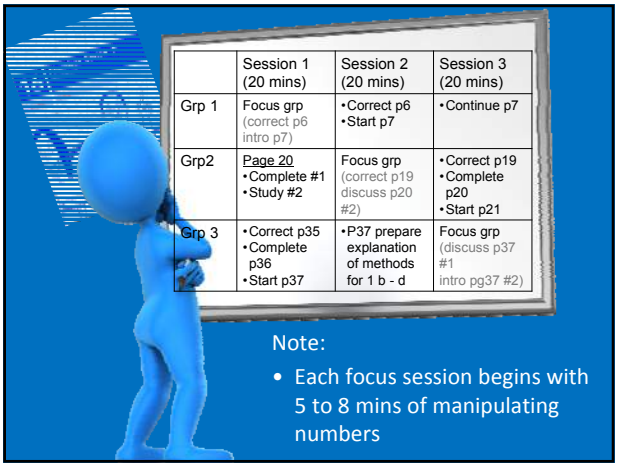
What kind of counting activity/activities will I do on the mat to prepare the children to complete this task?
Refer to handout for counting activity names!

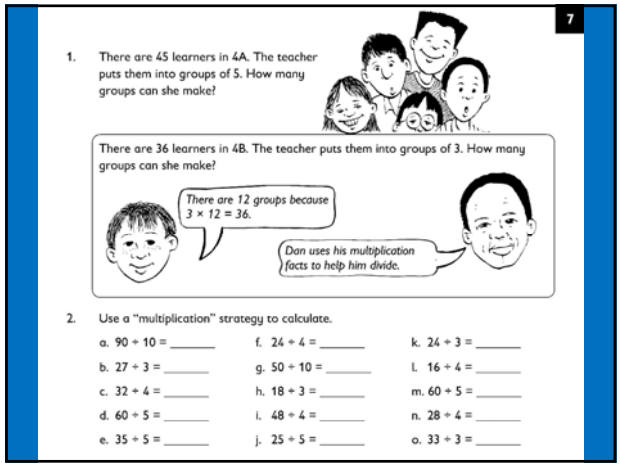
What kind of manipulating number activity/activities will I do on the mat to prepare the children to complete this task?
Refer to handout for manipulating number activity names!

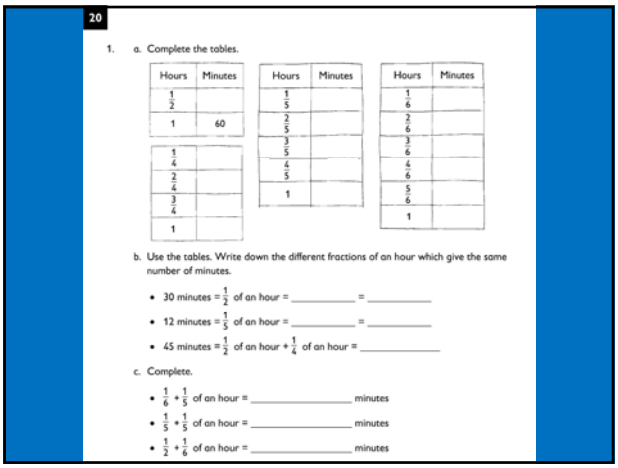
What kind of problem will I do on the mat to prepare the children to complete this task?
Refer to handout for problem types!

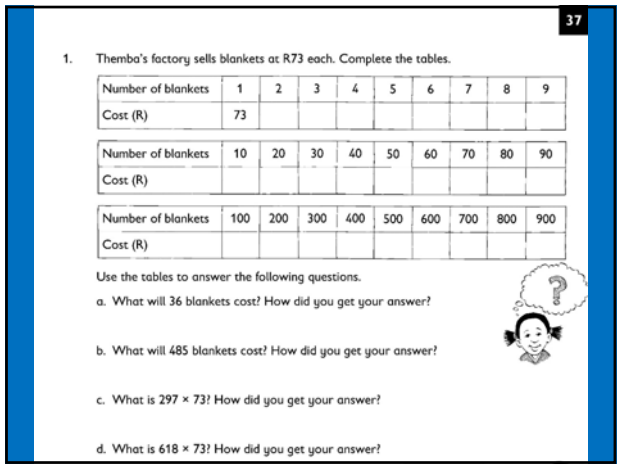




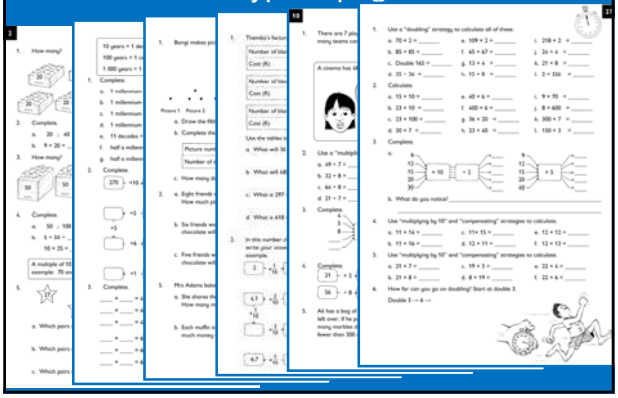








A typical page



Independent learning – the role of the child

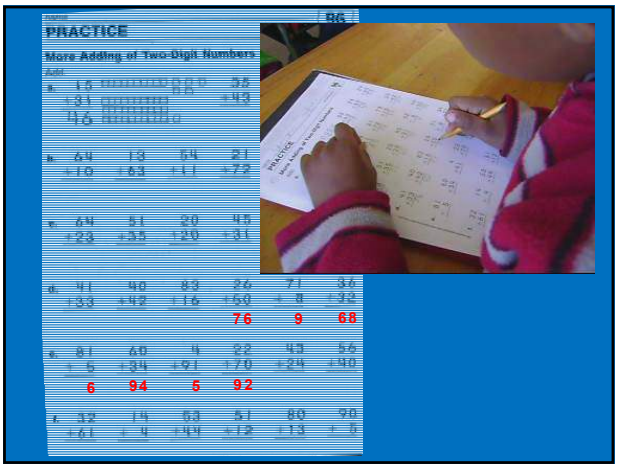
The Number Sense Workbooks

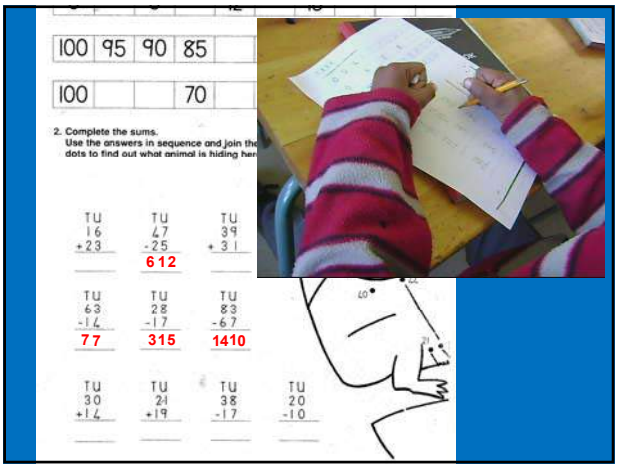
- In the early grades (grades 1 to 3)
  - The workbooks are used to consolidate the focus group session, i.e. the activity reflects the experience of the session and the children work independently to complete the "page of the day". After completing the page the teacher will facilitate reflection,
- In the later grades (grades 4+)
  - Children complete pages in the workbooks in preparation for their focus group session. Because children have typically "seen/done" each page previously they should be able to make a start. The focus group session is used to reflect on the activity.

Experiencing mathematics as a sense making activity

Mathematics is:
The memorisation of facts, rules, formulas and procedures needed to determine the answers to questions.

Mathematics is:
A meaningful, sense-making, problem solving activity.





1. Workers at the factory are paid R45 per hour.

Complete the tables to make it easy to work out how much they must be paid for a specific number of hours.

Number of hours	1	2	3	4	5	6	7	8	9
Pay (R)	45	90	135	180	225	270	315	360	405

Number of hours	10	20	30	40	50	60	70	80	90
Pay (R)	450								

Number of hours	100	200	300	400	500	600	700	800	900
Pay (R)									

Number of hours	1	2	3	4	5	6	7	8	9
Pay (R)	45	90	135	180	225	270	315	360	405

Number of hours	10	20	30	40	50	60	70	80	90
Pay (R)	450		1350						

Number of hours	100	200	300	400	500	600	700	800	900
Pay (R)	4500								

Use the tables to answer the following questions.

a. Tim works 136 hours this month. How much will he be paid?

b. Manare worked for 209 hours. How much will he be paid?

c. What is 45 × 28?

d. What is 45 × 109?

136 × 45 =

136 = 100 + 30 + 6
100 × 45 = 4 500
30 × 45 = 1 350
6 × 45 = 270

4 500 + 1 350 + 270

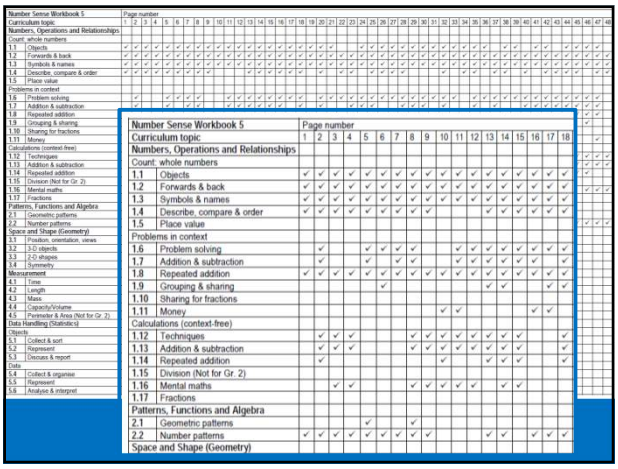
$$\begin{array}{r} {}^{3}{}^{1}45 \\ \times\ 136 \\ \hline 270 \\ 1350 \\ 4500 \\ \hline \end{array}$$

6 × 5 = 30
6 × 4 = 24
24 + 3 = 27

In groups:

- Make a list of questions that you would like answered in terms of "making the most of the Number Sense Workbooks"
- Discuss any solutions that you have to the questions raised within the experience of the group
- Be ready to participate in a feedback session

The Number Sense Workbooks and the curriculum



Number Sense Workbook 5		Page number																
Curriculum topic		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Numbers, Operations and Relationships																			
Count: whole numbers																			
1.1	Objects	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
1.2	Forwards & back	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
1.3	Symbols & names	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
1.4	Describe, compare & order	✓	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓	✓	✓	✓	✓	✓
1.5	Place value																	✓	✓
Problems in context																			
1.6	Problem solving		✓			✓	✓	✓	✓			✓	✓	✓	✓	✓	✓		✓
1.7	Addition & subtraction		✓			✓		✓	✓			✓	✓	✓	✓	✓			✓
1.8	Repeated addition	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
1.9	Grouping & sharing						✓							✓	✓			✓	✓
1.10	Sharing for fractions																		
1.11	Money										✓	✓					✓	✓	
Calculations (context-free)																			
1.12	Techniques		✓	✓	✓			✓	✓	✓	✓	✓	✓	✓	✓				✓
1.13	Addition & subtraction		✓	✓	✓			✓	✓	✓	✓	✓	✓	✓	✓				✓
1.14	Repeated addition		✓								✓			✓	✓				✓
1.15	Division (Not for Gr. 2)																		
1.16	Mental maths			✓	✓			✓	✓	✓	✓	✓		✓	✓				
1.17	Fractions																		
Patterns, Functions and Algebra																			
2.1	Geometric patterns					✓		✓											
2.2	Number patterns	✓	✓	✓	✓	✓	✓	✓	✓					✓	✓		✓	✓	✓
Space and Shape (Geometry)																			

Problem solving:

Knowing what to do when you don't know what to do!

Problem solving:

- Problems as exercise
  - A task is used to introduce a technique
  - The technique is illustrated
  - More tasks are provided so that the student may practice the illustrated skills

Problem solving:

- Problem solving as a context
  - Justification for teaching mathematics (problems related to the real world)
  - To provide motivation for subject topics
  - As recreation – "mathematics is fun"
  - As a means for developing new skills
  - As practice
- Problem solving as skill
  - Problem solving is taught as a skill (routine and non-routine problems)
- Problem solving as an art
  - "Doing mathematics" – solving perplexing problems – students should engage in problems solving, learning to solve problems of significant difficulty and complexity
  - Mathematics as a sense-making activity
  - Seeing phenomena in mathematical terms

Problem solving – the role of the teacher

Teaching actions for problem solving

BEFORE

Teaching action	Purpose
• Read the problem – discuss words or phrases students may not understand	• Illustrate the importance of reading carefully: focus on special vocabulary
• Use whole-class discussion to focus on importance of understanding the problem	• Focus on important data, clarification process
• (Optional) Whole-class discussion of strategies to solve the problem	• Elicit ideas for possible ways to solve the problem

Teaching actions for problem solving

DURING

Teaching action	Purpose
• Observe and question students to determine where they are	• Diagnose strengths and weaknesses
• Provide hints as needed	• Help students past blockages
• Provide problem extensions as needed	• Challenge early finishers to generalise
• Require students who obtain a solution to "answer the question"	• Require students to look over their work and make sure it makes sense

Teaching actions for problem solving

AFTER

Teaching action	Purpose
• Show and discuss solutions	• Show and name the different strategies
• Relate to previously solved problems or have students solve extension problems	• Demonstrate the general applicability of problem solving strategies
• Discuss special features	• Show how features of the problem may influence the approach

Basic steps of problem solving

- Find out
- Choose a strategy
- Solve it
- Look back

- Find out
  - Read the problem carefully
  - Have you done a similar problem before?
  - How is this problem similar?
  - How is it different?
  - What facts do you have?
- Choose a strategy
- Solve it
- Look back

- Find out
- Choose a strategy
  - What strategies do you know?
  - Try a strategy that seems as if it will work.
  - If it doesn't help you, it may lead you to one that will.
- Solve it
- Look back

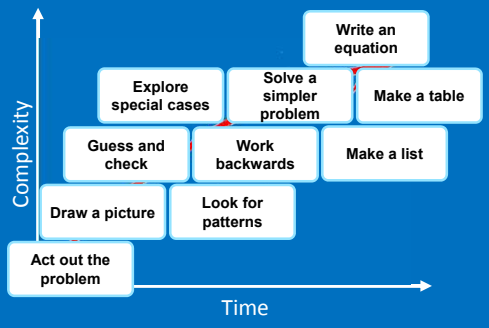
Some strategies

- Calculate or simplify
- Use a formula
- Make a model or draw a diagram
- Make a table, list or chart
- Guess, check and revise
- Consider a simpler case
- Eliminate
- Look for a pattern

- Find out
- Choose a strategy
- Solve it
  - Use the strategy you selected and work on the problem.
- Look back

- Find out
- Choose a strategy
- Solve it
- Look back
  - Read the question again.
  - Did you answer the question asked?
  - Is your answer in the correct units?
  - Does your answer seem reasonable?

Problem solving strategies



Question that support the development of problem solving strategies

- What do you know, what do you want to know? What don't you know?
- What do you notice:
  - What is the same and what is different?
- Have we solved a similar problem before – how did we solve that?
- Can you: draw a picture; make a list; make a table; see a pattern; try something…

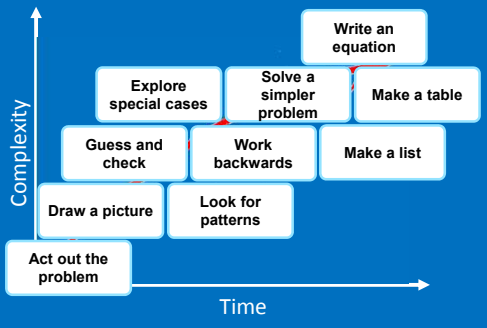
A "product of a number" is the product of the numbers that sum to the number

Illustration: consider the number 8
since 5 + 3 = 8, 5 × 3 = 15 is a product of 8
since 5 + 2 + 1 = 8, 5 × 2 × 1 = 10 is a product of 8

What is the "largest product" of 20?

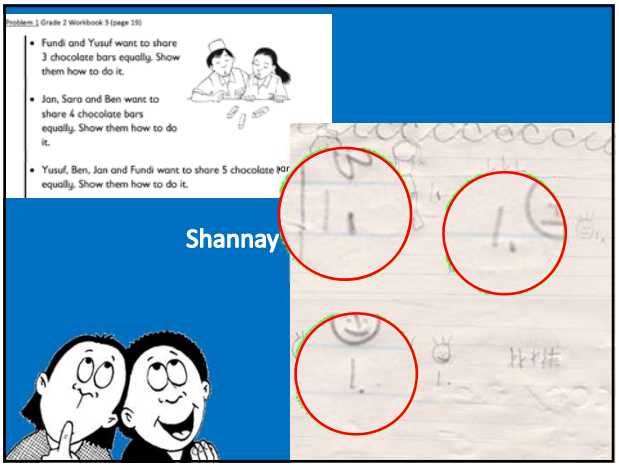
Sam has more than 30 marbles.
When he arranges the marbles in piles of 5 there is 1 marble left over.
When he arranges the marbles in piles of 7 there are 3 marbles left over.
How many marbles does Sam have?

Problem solving strategies



Problem 1 Grade 2 Workbook 3 (page 19)

- Fundi and Yusuf want to share 3 chocolate bars equally. Show them how to do it.
- Jan, Sara and Ben want to share 4 chocolate bars equally. Show them how to do it.
- Yusuf, Ben, Jan and Fundi want to share 5 chocolate bars equally. Show them how to do it.



Problem 6 Grade 3 Workbook 2 (page 14)

- Four children share 13 chocolate bars equally. How much chocolate will each child get?
- Six children share 7 chocolate bars equally. How much chocolate will each child get?

Problem 7 Grade 3 Workbook 2 (page 24)

- Three children share 5 chocolate bars equally. Show how they must do it.
- Four children share 6 chocolate bars equally. Show how they must do it.

In grade groups:

- Study the workbooks appropriate to your grade and identify examples of problems that are:
  - Developing new skills
  - Engaging children in "doing mathematics" – solving perplexing problems

The Number Sense Workbooks and the role of routines

Having a *Sense of Number* means:

- Being able to work fluently and flexibly with numbers and number concepts.
- Having a rich understanding of the meaning of number.
- Having a wide range of effective strategies for solving a large variety of number problems.

Strategies (Australian Growth points) Addition and Subtraction:

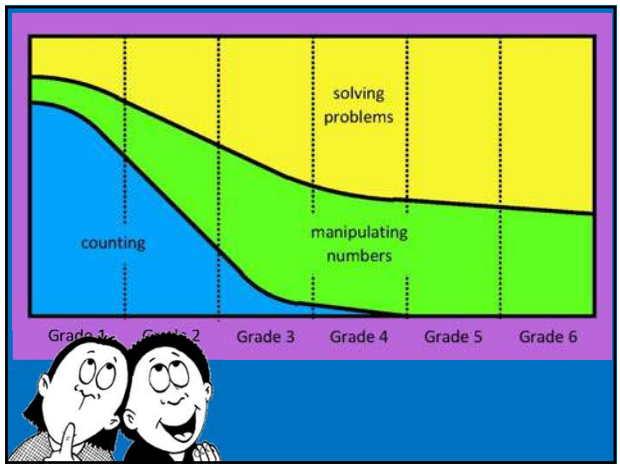
0. Not apparent.
   - *Not yet able to combine and count two collections of objects.*
1. Count all (two collections)
   - *Counts all to find the total of two collections.*
2. Count on
   - *Counts on from one number to find the total of two collections.*
3. Count back/count down to/count up from
   - *Given a subtraction situation, chooses appropriately from strategies including count back, count down to and count up from.*

4. Basic strategies (doubles, commutativity, adding 10, tens facts, other known facts)
   - *Given an addition or subtraction problem, strategies such as doubles, commutativity, adding 10, tens facts, and other known facts are evident.*
5. Derived strategies (near doubles, adding 9, build to next ten, fact families, intuitive strategies)
   - *Given an addition or subtraction problem, strategies such as near doubles, adding 9, build to next ten, fact families and intuitive strategies are evident.*
6. Extending and applying addition and subtraction using basic, derived and intuitive strategies
   - *Given a range of tasks (including multi-digit numbers), can solve them mentally, using the appropriate strategies and a clear understanding of key concepts*
7. Extending and applying addition and subtraction strategies – fractions and decimals

Strategies (Australian Growth points) Multiplication and Division:

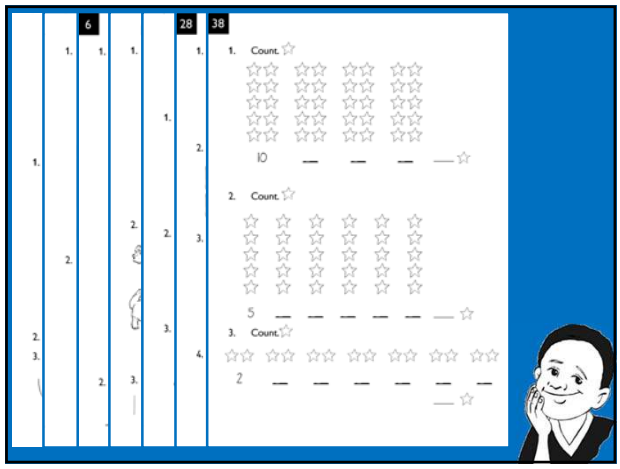
0. Not apparent.
   - *Not yet able to create and count the total of several small groups.*
1. Counting group items as ones
   - *To find the total in a multiple group situation, refers to individual items only.*
2. Modelling multiplication and division (all objects perceived)
   - *Models all objects to solve multiplicative and sharing situations.*
3. Partial modelling multiplication and division (some objects perceived)
   - *Solves multiplication and division problems where objects are not all modelled or perceived.*
- Abstracting multiplication and division (no objects

4. Abstracting multiplication and division (no objects perceived)
   - *Solves multiplication and division problems where objects are not modelled or perceived.*
5. Basic derived and intuitive strategies for multiplication
   - *Can solve a range of multiplication problems using strategies such as commutativity and building up from known facts.*
6. Basic, derived and intuitive strategies for division
   - *Can solve a range of division problems using strategies such as fact families and building up from known facts.*
7. Extending and applying multiplication and division
   - *Can solve a range of multiplication and division problems (including multi-digit numbers) in practical contexts*
8. Extending and applying multiplication and division – fractions and decimals



Counting activities

- Rote counting activities:
  - Counting in ones
  - Counting rhymes and songs
  - Counting is steps
- Rational counting activities:
  - Counting small sets of counters in ones
  - Counting out small groups of counters
  - Estimating and counting larger sets of counters in ones
  - Counting in groups
  - Counting large sets of counters in groups



Manipulating number activities

- Single digit arithmetic
- Arithmetic with multiples of ten, hundreds and thousands
- Completing tens (hundreds and thousands) including adding and subtracting to and from multiples of ten
- Bridging tens (hundreds and thousand)

- Doubling and halving – to develop efficient division and multiplication strategies
- Interrelated multiplication facts

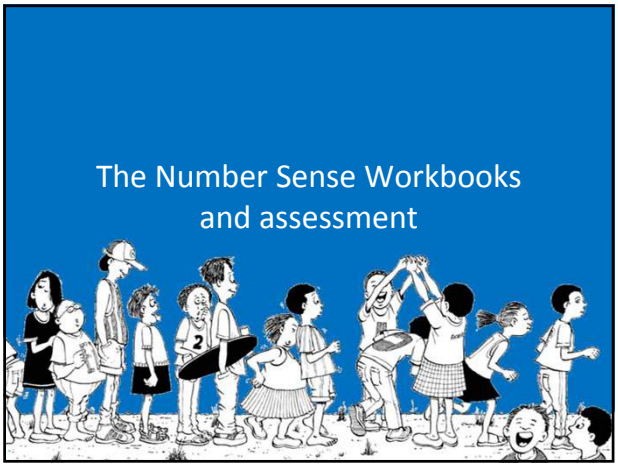
Solving problems

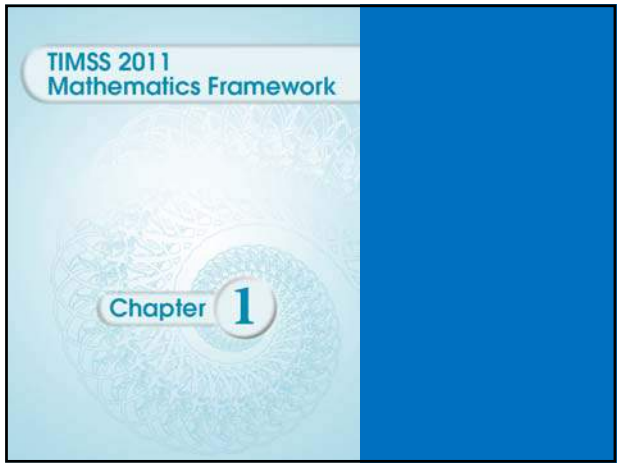
- Problems that support the development of addition and subtraction including:
  - Change, Combine, and Compare problems
- Problems that support the development of the division concept:
  - Sharing, and Grouping
- Problems that support the development of the multiplication concept:
  - Repeated addition, and Situations with a grid or array type structure
- Problems that support the development of the following concepts:
  - Fractions, Ratio, rate and proportion including sharing in a ratio.

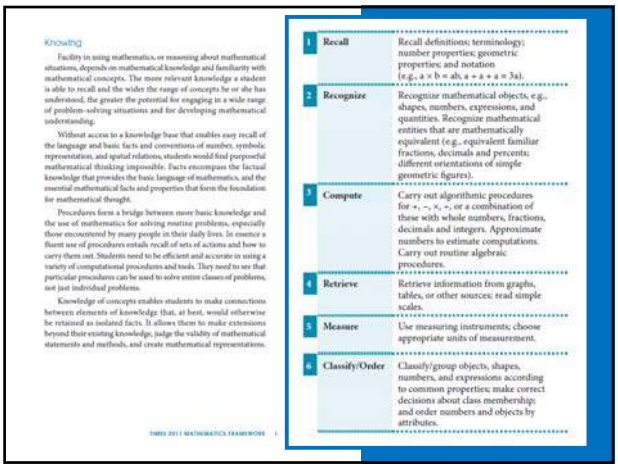
In grade groups:

- Study the workbooks appropriate to your grade and discuss how the workbook pages:
  - Support children to work fluently and flexibly with numbers and number concepts.
  - Develop a wide range of effective strategies for solving a large variety of number problems.

Support for the Number Sense Workbooks







Reasoning

Reasoning mathematically involves the capacity for logical, systematic thinking. It includes intuitive and inductive reasoning based on patterns and regularities that can be used to arrive at solutions to non-routine problems. Non-routine problems are problems that are very likely to be unfamiliar to students. They make cognitive demands over and above those needed for solution of routine problems, even when the knowledge and skills required for their solution have been learned. Non-routine problems may be purely mathematical or may have real-life settings. Both types of items involve transfer of knowledge and skills to new situations, and interactions among reasoning skills are usually a feature. Problems requiring reasoning may do so in different ways, because of the novelty of the context or the complexity of the situation, or because any solution to the problem must involve several steps, perhaps drawing on knowledge and understanding from different areas of mathematics.

Even though of the many behaviors listed within the reasoning domain are those that may be drawn on in thinking about and solving novel or complex problems, each by itself represents a valuable outcome of mathematics education, with the potential to influence learners' thinking more generally. For example, reasoning involves the ability to observe and make conjectures. It also involves making logical deductions based on specific assumptions and rules, and justifying results.

1	**Analyze**	Determine, describe, or use relationships between variables or objects in mathematical situations, and make valid inferences from given information.
2	**Generalize/ Specialize**	Extend the domain to which the result of mathematical thinking and problem solving is applicable by restating results in more general and more widely applicable terms.
3	**Integrate/ Synthesize**	Make connections between different elements of knowledge and related representations, and make linkages between related mathematical ideas. Combine mathematical facts, concepts, and procedures to establish results, and combine results to produce a further result.
4	**Justify**	Provide a justification by reference to known mathematical results or properties.
5	**Solve Non-routine Problems**	Solve problems set in mathematical or real life contexts where students are unlikely to have encountered closely similar items, and apply mathematical facts, concepts, and procedures in unfamiliar or complex contexts.

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