GRADE

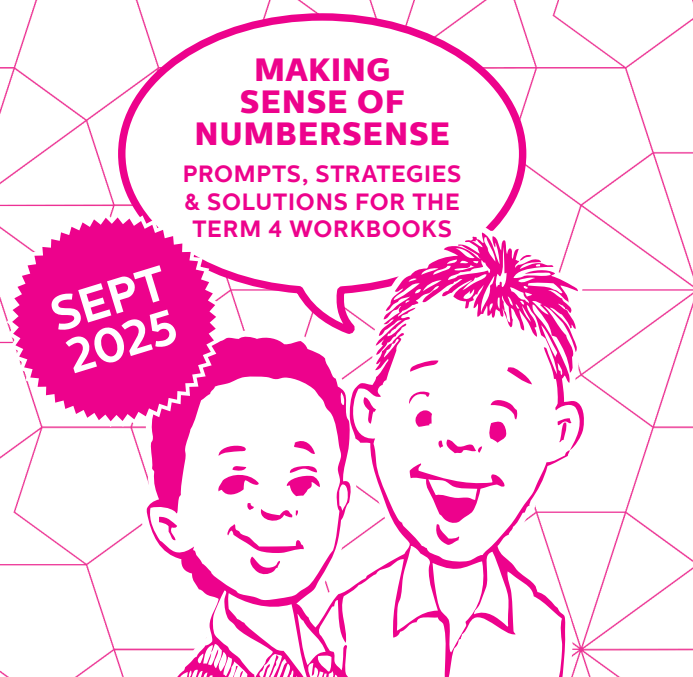
R

NumberSense

PROMPTS, STRATEGIES & SOLUTIONS

English

Teacher's Guide



ABOUT THIS TEACHER GUIDE

The NumberSense Mathematics Programme aims to make the teaching and learning of mathematics an integrated, sense-making, problem-solving activity. By providing useful prompts, strategies and solutions, teachers are empowered to make sense of the NumberSense Workbooks and resources within their context. The aim of this guide is to support teachers in provoking reflective discussions where reasoning is applied so that connections between activities and solution strategies can be developed.

Please note that this is a sample, draft version, which includes the start of a comprehensive Grade R Teacher Guide.

The following is included:

1. Using NumberSense in Grade R

This section includes:

a) Curriculum mapping: a year plan showing NumberSense resources and content coverage over the course of the year as well as how this corresponds to the CAPS curriculum.

b) NumberSense approach to Grade R: Grade R-focussed teaching and learning strategies and structures

c) Planning resources: photocopiable planning tools (a one-page planning prompt with simple planning prompts and a reflection question bank).

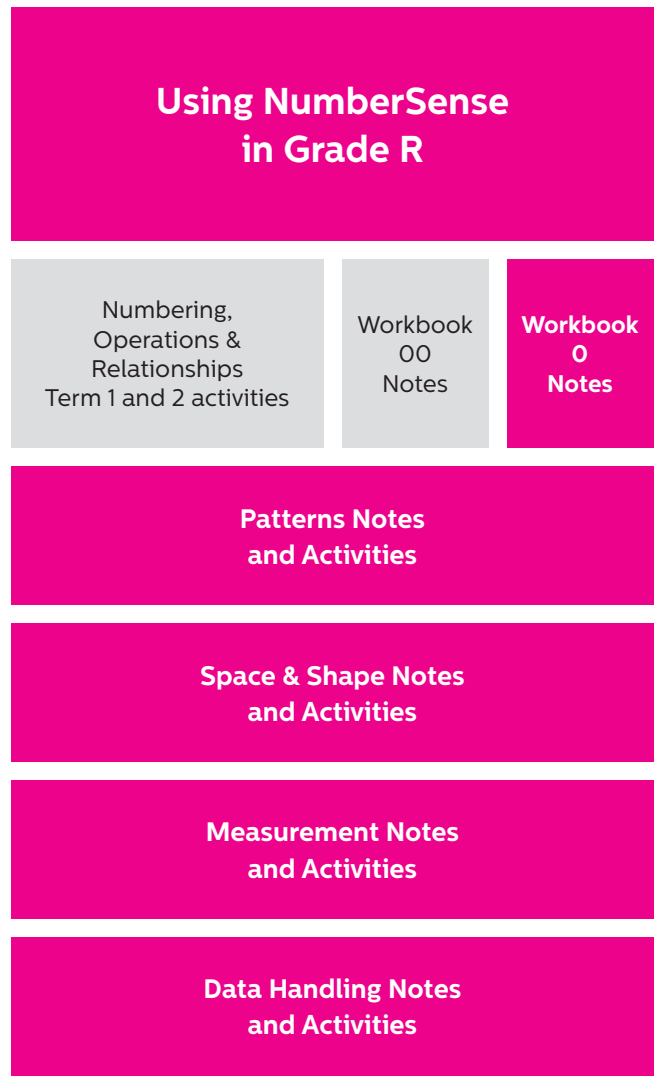
2. Workbook support

This section includes:

a) Number concept activities for Term 1 and 2. (Note: this section is under development and not included at present).

b) Written prompts and teaching strategies for teachers on key pages of Workbook 00 and 0 (Term 3 and 4) (only Workbook 0 included at present), together with solutions, references to similar activities on other pages, and more detailed 'things to think about' pages which explain the reasoning behind the activities and give ideas and suggestions for lessons.

c) Additional information regarding the conceptual development of the Pattern, Space & Shape, Measurement and Data Handling content areas as well as activity suggestions.



CURRICULUM MAPPING

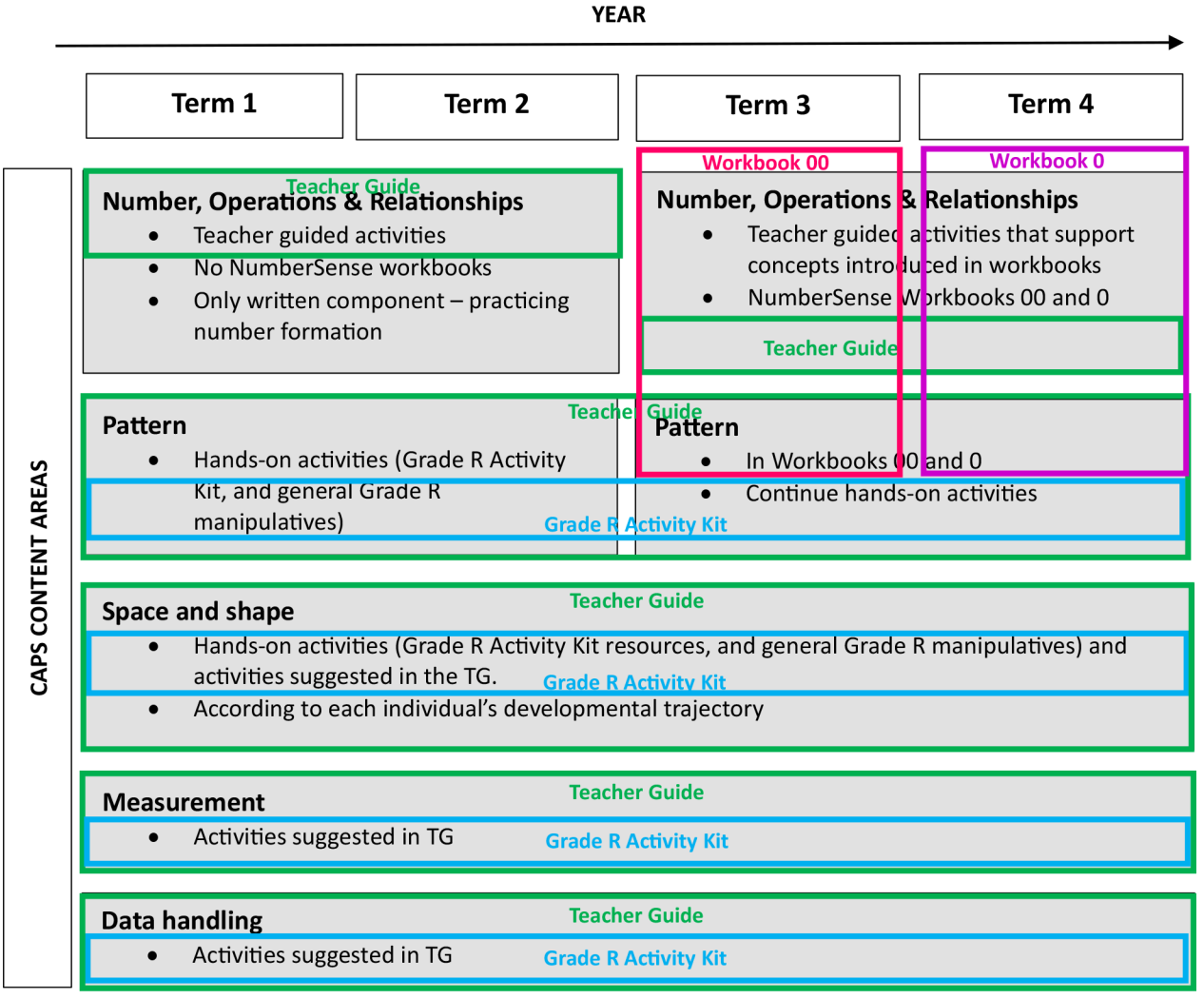
GRADE R WORKBOOK CURRICULUM COVERAGE

			TERM 1	TERM 2	TERM 3	TERM 4
					WORKBOOK 00	WORKBOOK 0
NUMBERS, OPERATIONS & RELATIONSHIPS, PATTERNS						
COUNT: WHOLE NUMBERS	1.1	Objects			✓	✓
	1.2	Forwards & back			✓	✓
	1.3	Symbols & names			✓	✓
	1.4	Describe, compare & order			✓	✓
	1.5	Place value				
PROBLEMS IN CONTEXT	1.6	Problem solving			✓	✓
	1.7	Addition & subtraction			✓	✓
	1.8	Repeated addition			✓	✓
	1.9	Grouping & sharing			✓	✓
	1.10	Sharing for fractions				
	1.11	Money				
CALCULATIONS (CONTEXT-FREE)	1.12	Techniques			✓	✓
	1.13	Addition & subtraction			✓	✓
	1.14	Repeated addition				✓
	1.15	Division				✓
	1.16	Mental maths			✓	✓
	1.17	Fractions				
PATTERNS FUNCTIONS	2.1	Geometric patterns			✓	✓
	2.2	Number patterns				
SPACE & SHAPE (GEOMETRY)						
	3.1	Position, orientation, views				✓
	3.2	3-D objects				
	3.3	2-D shapes			✓	✓
	3.4	Symmetry				
MEASUREMENT						
	4.1	Time			✓	✓
	4.2	Length			✓	
	4.3	Mass				
	4.4	Capacity/Volume				
	4.5	Perimeter & Area				
DATA HANDLING						
OBJECTS	5.1	Collect & sort				
	5.2	Represent				
	5.3	Discuss & report				
DATA	5.4	Collect & organise			✓	
	5.5	Represent				
	5.6	Analyse & interpret				

USING NUMBERSENSE IN GRADE R

The NumberSense Grade R workbooks are designed to inspire activities that are play-based and centred around concrete activities. There are only two workbooks as the intention is that teachers only start to use the workbooks in the second or third term of the year. Prior to that, the teacher can use the workbooks as a guide to creating meaningful mathematics activities, problems and situations. The workbooks therefore serve primarily as an activity guide and do not replace concrete work that should be done with counters, beads and other manipulatives.

Curriculum mapping



This diagram illustrates the NumberSense Mathematics Programme resources that will be available for the different areas of the CAPS Mathematics Content Areas and how these resources correspond to the different terms. Grade R is unique in that NumberSense does not promote the use of workbooks in the first one to two terms of the year. Grade R focuses more on hands-on concrete activities than the later grades. Many sections of the curriculum are best explored only at a concrete level.

NumberSense Mathematics Programme resources

Workbook 00 and 0

If and when children do begin working in the workbooks, it is imperative that the teacher mediates the activities. The activity in the workbook should only be done once a similar concrete activity has been done on the mat or in small groups. The workbooks thereby provide support and consolidation of play-based, concrete activities.

Children should not be expected to work independently in the workbooks; rather, the teacher should facilitate all the learning activities. The children should be encouraged to make sense of the situations and not simply be told how to fill in the page. This is achieved through reflective questioning. Children should be asked to explain their answers, describe any patterns they have observed, and ask questions of their own.

The pages should be done in sequence as each page builds on the previous pages in a spiralled manner, introducing and reintroducing mathematical concepts, continually building on prior instruction.
Teachers should become comfortable knowing that some children may not 'get it' the first or even the second time. However, they can be confident that through further play with concrete apparatus and continued reflective questioning, the children will begin to develop confidence and a robust sense of number.

Due to the mediated nature of the Grade R workbook activities, it is not expected that the Grade R children work independently. The activities in Workbooks 00 and 0 may appear more complex in nature than those in the first half of Workbook 1. This is because in Workbook 1 the expectation is that the children complete the page independently.

The skills and knowledge underpinning the activities in each Grade R workbook are: rote counting, rational counting, reading and writing numbers, comparing quantities and solving problems in context.
It is recommended that teachers pace the workbooks according to the developmental level of the children in the class, repeating activities as needed but also continuing with new activities. Teachers can do this with the confidence that the same mathematical concepts will be covered again and again.

Teacher Guide (under development)

The completed Teacher Guide will provide practical activity ideas and guidance to support the activities in Workbook 00 and 0 as outlined above. It will also contain activity ideas for Number, Operations and Relationships for Term 1 and 2, as well as activities ideas for the whole year for Pattern, Space and Shape, Measurement and Data Handling.

Grade R Activity Kit (under development)

The completed Grade R Activity Kit will contain Grade R appropriate activity cards to be used with materials such as Mosaic Puzzles, Attribute Blocks, Tangram Puzzles and connecting cubes.

Additional materials

It is expected that Grade R teachers will already have a collection of manipulatives and learning materials in their classrooms and there is no need for NumberSense to duplicate these materials.

These materials should include, but are not limited to:

- Counting manipulatives
- Blocks and construction toys
- Jigsaw puzzles
- Ice-cream sticks and/or match sticks
- Pegs and peg boards
- Beads
- Playdough
- 3D recyclable objects
- Sand and water play equipment
- Calendar resources
- Art resources (2D and 3D)

The use of these additional materials explains why the NumberSense Grade R Activity Kit does not overlap completely with the NumberSense Teacher Guide in the year overview diagram.
It is anticipated that Grade R children will require many more opportunities to practise number formation and writing than are presented in Workbook 00 and 0. Therefore, teachers will need to continue to use additional materials such as number tracing templates. (This is illustrated by the gap in overlap in Number, Operations and Relationships for Term 1 and 2).

The NumberSense Approach to Grade R

Conceptual understanding

Grade R teachers may notice as much as a four-year variation in the development of different skills amongst various children in their class. The aim is to accommodate for multiple levels of functioning, stimulating the children at the level they are at and gradually moving them through the levels to be age-appropriate by the end of Grade R. To do this, we have to be clear about what we want to achieve, where we are in the journey, where we are going (plan forwards) and where we have come from (plan backwards). Learning trajectories can help us to understand the specific developmental building blocks in different skill areas. Some learning trajectories will be included in the next sections about the mathematics content areas.

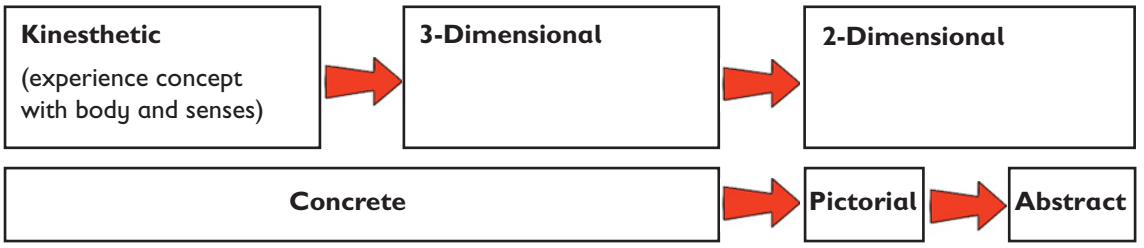
In Grade R, there should be a focus on the development of pre-mathematics skills such as:

- matching
- sorting
- comparing
- classifying
- ordering
- problem solving
- providing good reason
- communication
- making connections
- representation

These skills are heavily reliant on the individual child's development of underlying skills in:

- visual perception
- auditory perception
- language
- fine motor
- gross motor
- sensory processing

A solid early mathematical foundation is built by developing key concepts first through exposure to kinesthetic and concrete activities before moving onto pictures and eventually more abstract representations.



In Grade R, the NumberSense Mathematics Programme encourages teachers to make extensive use of hands-on, play-like activities to introduce mathematical concepts before introducing workbook activities.

Play, language and the role of the teacher

CAPS recommends that the approach to learning mathematics in Grade R should be based on the principles of integration and play-based learning. All classroom activities can be envisioned on a continuum according to where the locus of control lies between the child/children and adult or teacher.

A variety of activities at different points on this continuum are used to develop the Grade R child's mathematical ability. Many concepts, particularly those in Pattern, Space and Shape, Measurement and Data Handling are more suited to being explored through play-based activities than formal adult-directed activities. This does not mean that mathematical concepts develop simply by allowing children lots of "free play". The role of the Grade R teacher is to purposefully and proactively plan and mediate play-based learning experiences that will help the children develop age-appropriate mathematical concepts.

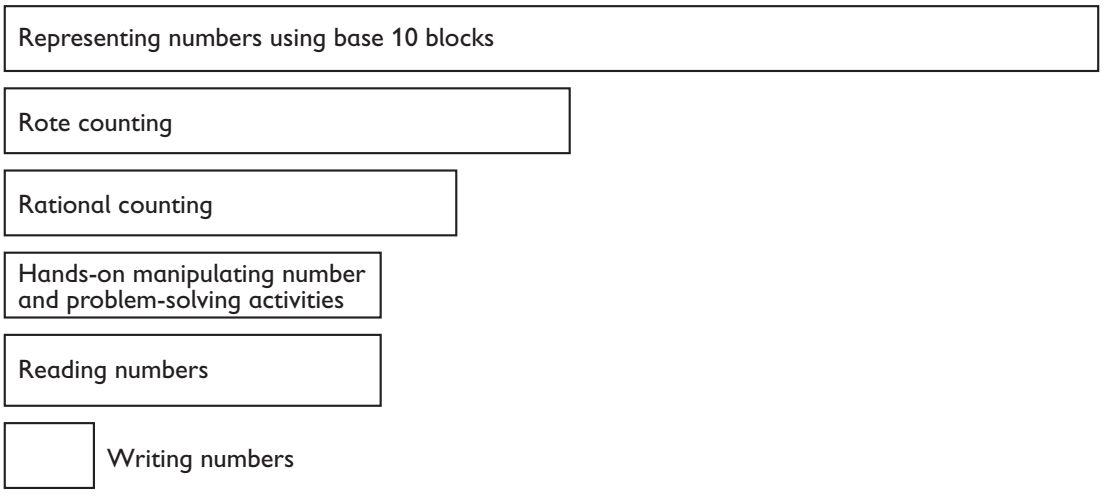
The role of the teacher

- Careful planning of activities according to learning trajectories, which may include less (closer to free play) or more structure, such as giving a challenge or task card.
- Asking questions to direct the children's attention to specific mathematical concepts or to provoke certain types of thinking.
- Introducing the children to the correct terminology and expanding their expressive language.
- Engaging children in discussion that encourages problem solving and the explanation of their thinking.
- Allowing opportunities for reflection and mediating these thoughts to advance their thinking.

Refer to the Question Bank page 40 for ideas of questions to stimulate children's thinking.

Number range

Young children benefit from exposure to visual representations of very large numbers to help develop their sense of number. Grade R teachers do not need to feel constrained to working within a number range that their children can write. Young children can represent and work with numbers at a higher range than they can write. Due to their developing visual-motor integration skills, a unique feature of Grade R children is that their ability to form number symbols on a page lags significantly behind their conceptualisation of numbers as represented by concrete objects and even by their ability to read number symbols.



A visual representation of how different number ranges can be used in different mathematical tasks.

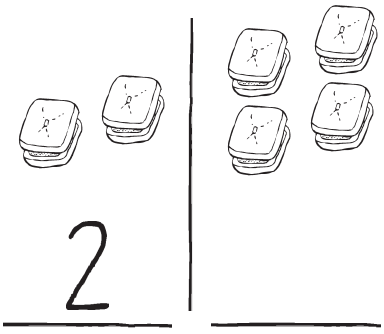
Note that context-free problem solving does not appear on this diagram as this level of abstract mathematical thinking is not expected at a Grade R level.

Notes

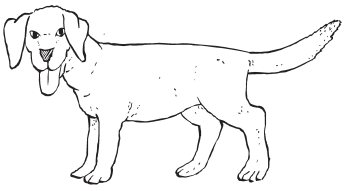
6

The big dog gets double what the small dog gets.

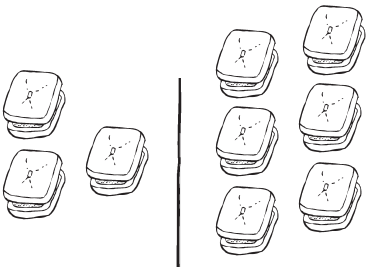
When the small dog gets 2 biscuits the big dog gets 4.



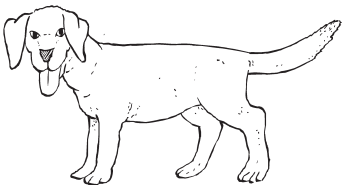
2 ____



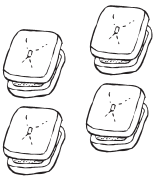
When the small dog gets 3 biscuits the big dog gets 6.



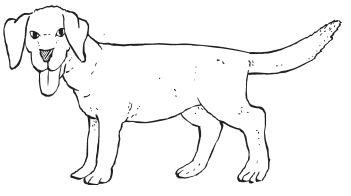
3 6



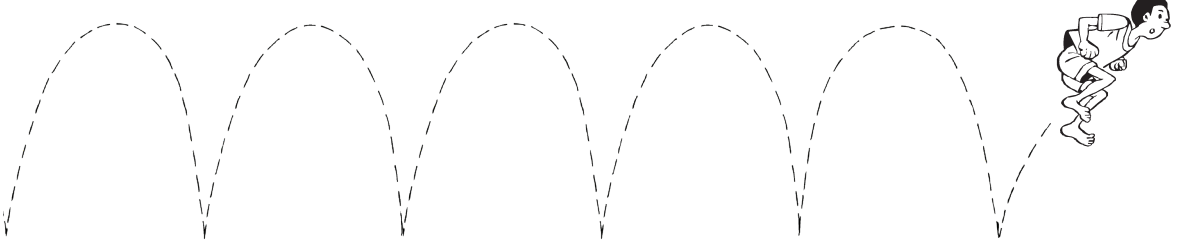
Double. Complete the picture. Write the number.



4 ____



Trace the hops.



Things to think about

Workbook 0
Page 6

Doubling problems ("big dog, small dog")

This is the first formal introduction to the concept of doubling.

Introducing the concept:

- Tell a story about the 2 dogs. Use props/models ensuring that 1 dog is clearly bigger than the other dog.
- Model a few examples:

? When the small dog gets 1 biscuit, the big dog gets 2 biscuits.
When the small dog gets 3 biscuits the big dog gets 6 biscuits.
What do you notice?
When we have 3 and then we get another 3, we call it "doubling".

The big dog gets double what the small dog gets.

Check for understanding (parallel problem) – as a class or group activity

Use numbers 10 and below.

? If the small dog gets 4 biscuits, how many will the big dog get?
How do you know? What is the pattern?

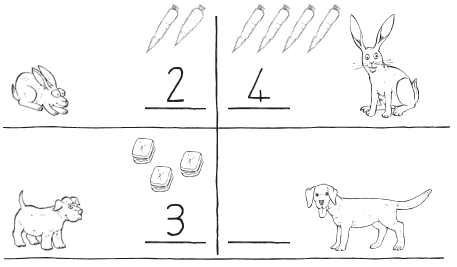
Children can be quite concerned about why the dogs get different amounts. This is one approach:

? Who has a baby brother or sister?
Does your baby brother or sister eat the same amount as you?

Once you are happy that the children understand the concept at a concrete level, allow them to try the problem in the workbook (p.6).

More practice:
Practise again the following day on the mat and in the workbook (p.7).
Even more practice:
Practise again 3 days later, on the mat and in the workbook (p.10).

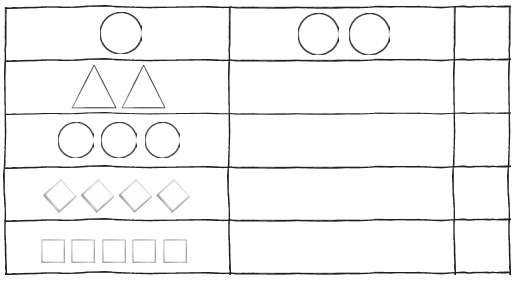
Double. Complete the picture .Write the number.



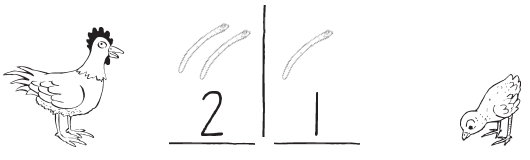
Further concept development:
On page 11, the concept of doubling becomes more abstract. Prepare children on the mat with concrete examples before letting them try the workbook.

The understanding of doubling forms a foundation for the concept of halving which is introduced on page 15.

Double.

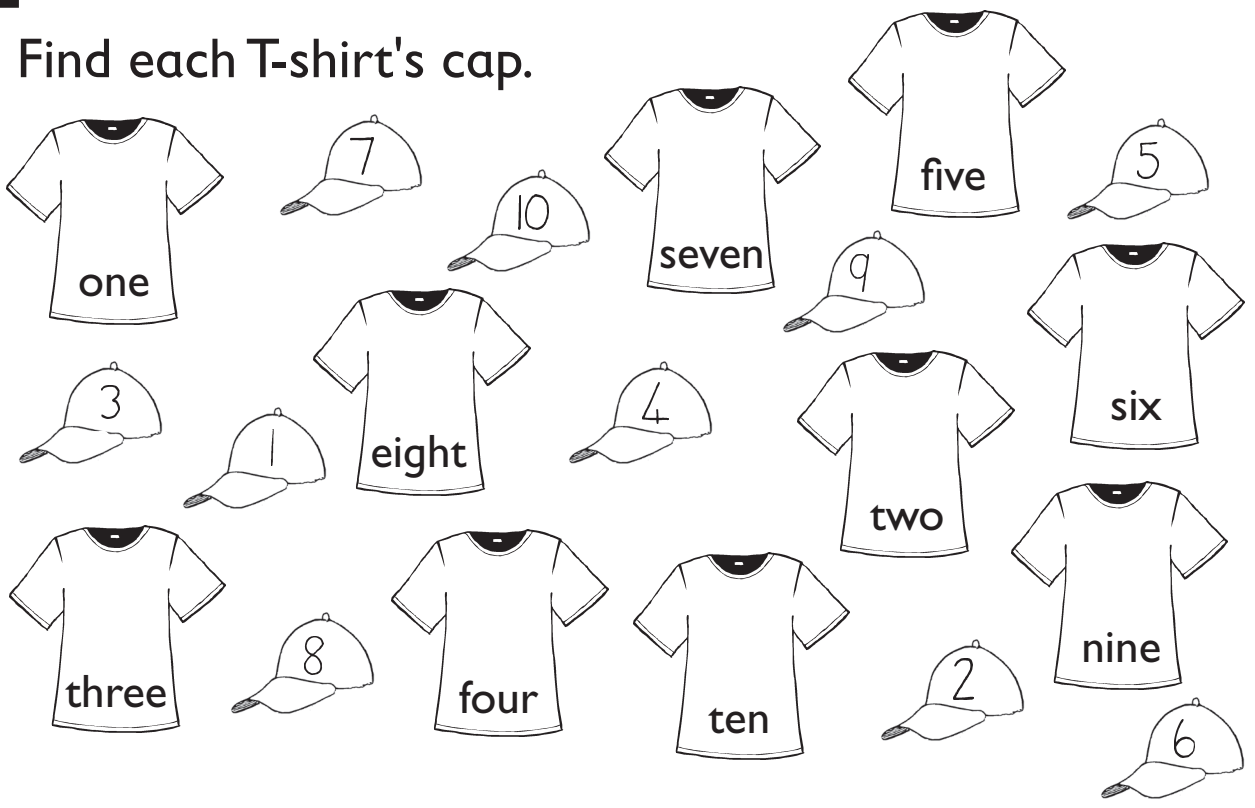


The small chicken gets half of what the large hen gets. When the large hen gets 2 worms the small chicken gets 1.

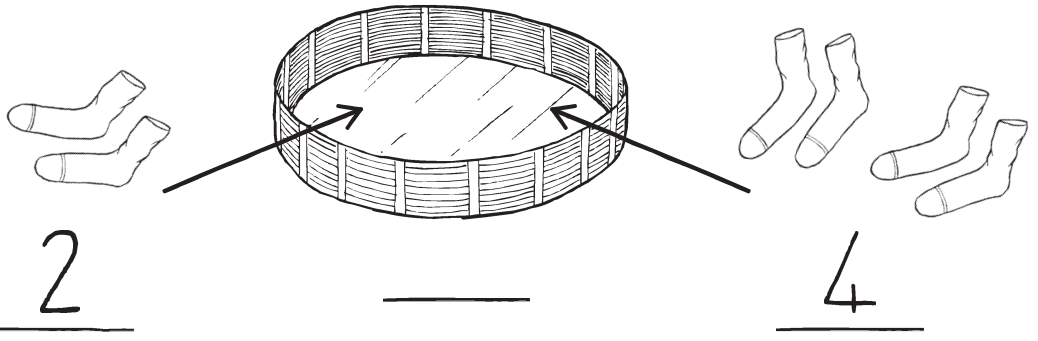


Workbook notes

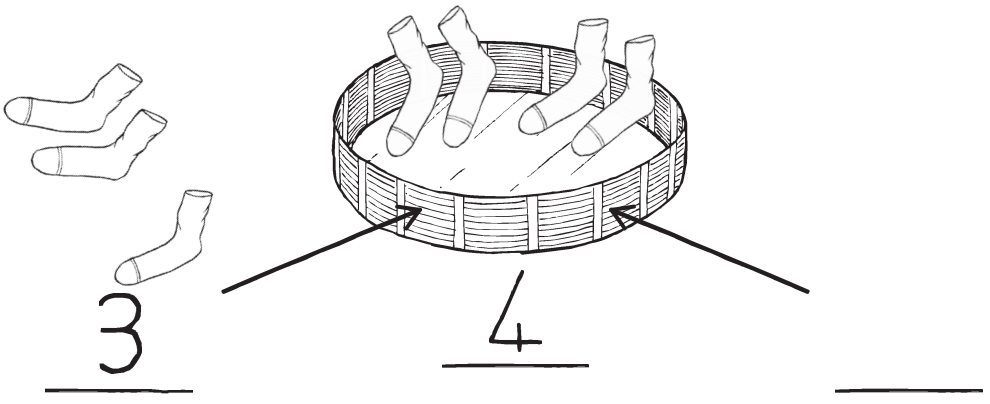
Find each T-shirt's cap.



Complete the picture. Write the number.



Complete the picture. Write the number.



Things to think about

Workbook 0
Page 8

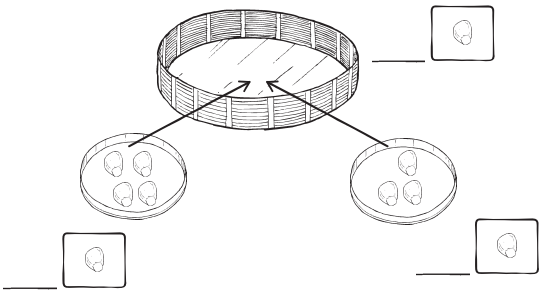
Addition and subtraction problems (missing addend)

First introduction to a related concept

Addition and subtraction are first introduced in Workbook 00 p.38 with children required to combine 2 quantities.

There are 3 more opportunities to practise (in a very similar style) in the book.

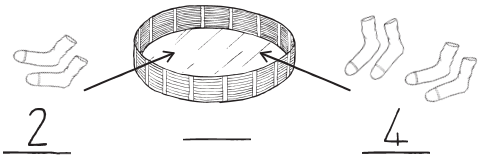
Put the mielies together. Write the number.



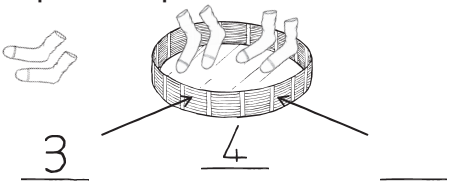
Now...

The addition concept changes slightly in Workbook 0 p.8 and introduces the problem of a missing addend.

Complete the picture. Write the number.



Complete the picture. Write the number.



Introducing the concept:

- Revise combining - use real socks, have 2 quantities (e.g. 2 and 4) and demonstrate putting both quantities in a basket.

? *How many socks are in the basket?*

- Now show the children 4 socks in the basket, cover the basket or put it behind your back and remove 1 sock without the children seeing how many socks were taken away. Ask the children to look at the basket again and tell you:

? *How many socks did I take away? How do you know?*

Check for understanding
(parallel problem)
– as a class or group activity
Use numbers 6 and below initially.

Once you are happy that the children understand the concept at a concrete level, allow them to try the problems in the workbook (p.8).

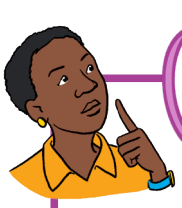
More practice:
Practise again the following day on the mat and in the workbook (p.9).

Even more practice:
There are many more workbook activities to practise combining, finding the missing addend, and writing the numbers in this workbook.

It is important to also keep practising at a concrete level (as part of your teacher-led, problem-solving time).

In Grade R, the equation format and symbols +, -, = do not need to be introduced.

Workbook notes



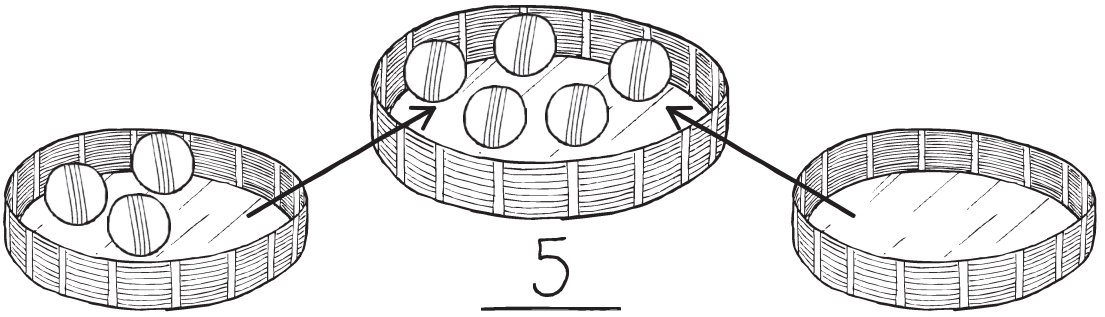
Refer to Workbook 0 p.8 to help you.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30

How many?

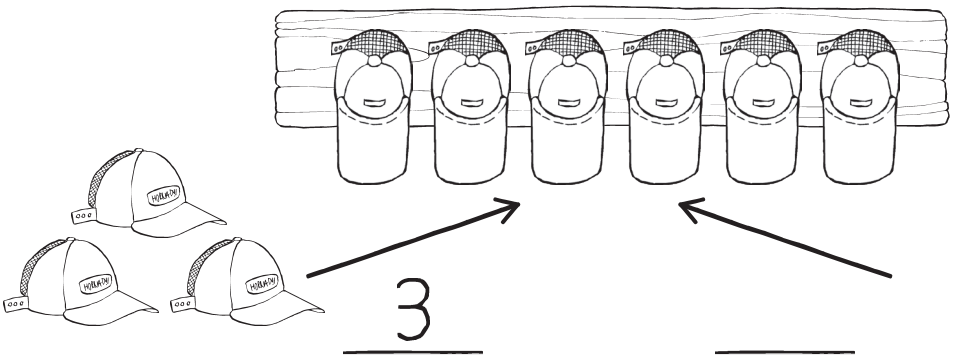
_____ 0

Complete the picture. Write the number.

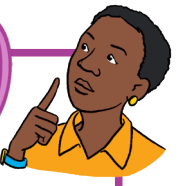


3 5 _____

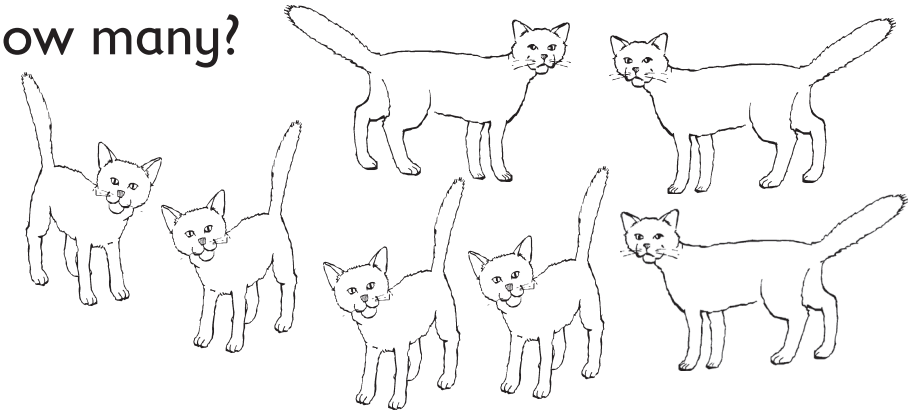
Complete the picture. Write the number.

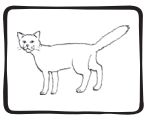


3 _____



How many?

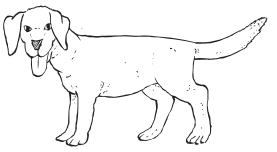


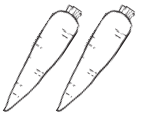


Double. Complete the picture . Write the number.

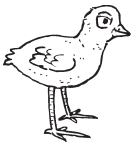
1

2

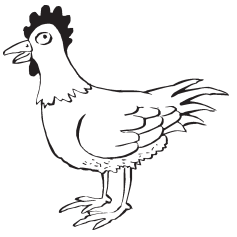


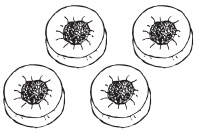


2



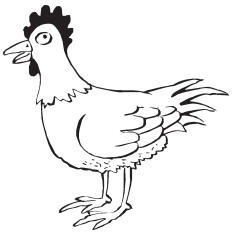
3





4

5



Workbook notes

Reflection questions for extension:

What did you notice about the number of cows and the number of legs?

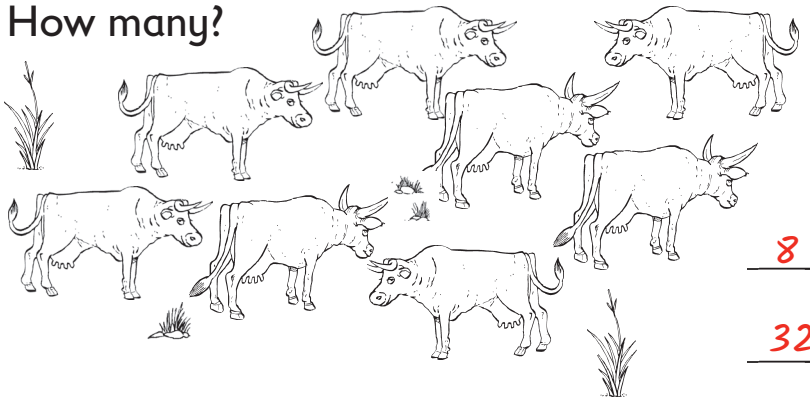
The number of legs is much bigger than the number of cows.

There are more legs than cows.

Why?

One cow has four legs.

How many?

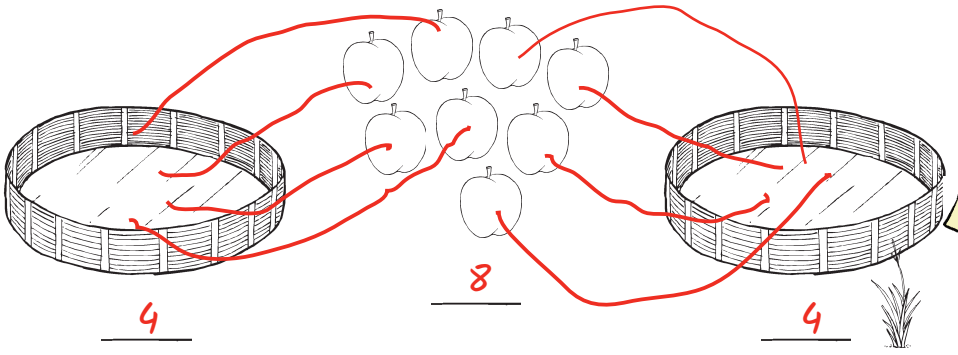


8

32

Check that children have an effective method of indicating which cows/legs they have counted and which ones still need to be counted.

Share the apples equally. Write the numbers.



4

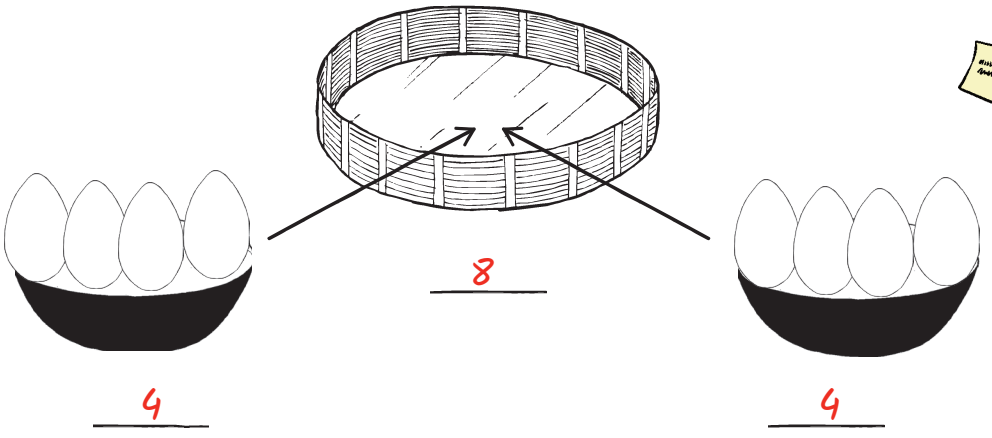
8

4

Refer to 'Things to think about' p. 27 in Workbook 00 where sharing problems are first introduced.

Check that children have an effective method of indicating which apples go into which basket (making sure that every apple has been allocated a basket).

Complete the picture. Write the numbers



4

8

4

Refer to the 'Things to think about' p. 28 in Workbook 00 where combining problems are first introduced.

Less confident children may need more practise at a concrete level.

Reflection questions for extension:

What do you notice about the two problems?

The numbers are the same.

I shared the apples into different baskets/ I put them into two groups.

What is the same? What is different?

I put all the eggs together/I put the two groups into one group.

I halved the apples. I doubled the eggs.

Reflection questions for halving in the table:

? What do you notice?

The big mouse always gets more than the small mouse.

The big mouse gets double what the small mouse gets.

The small mouse gets half of what the big mouse gets.

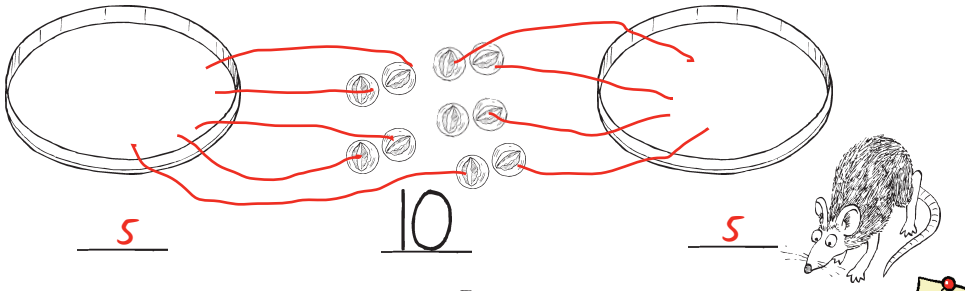
33

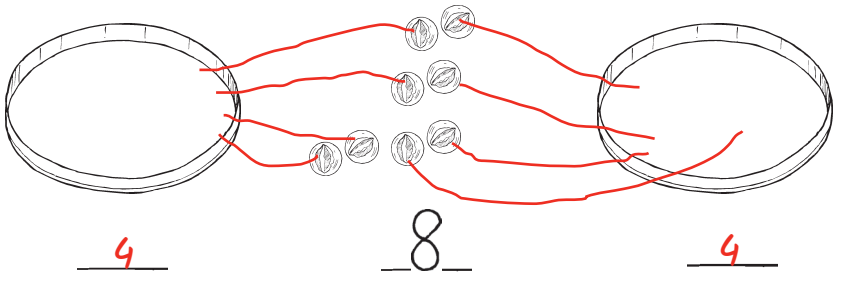
Halve.

		2
		1
		4
		3
		5

Refer to 'Things to think about' p. 6 where doubling problems are first introduced and p. 15 where halving problems are introduced.

Share the marbles equally. Write the numbers.





Refer to 'Things to think about' p. 27 in Workbook 00 where sharing problems are first introduced.

Check that children have an effective method of indicating which marbles go into which basket (making sure that every marble has been allocated to a basket).

Less confident children may need more practise at a concrete level.

Reflection questions for extension:

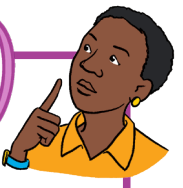
? What do you notice about the numbers?

5 is half of 10 and 10 is double 5.

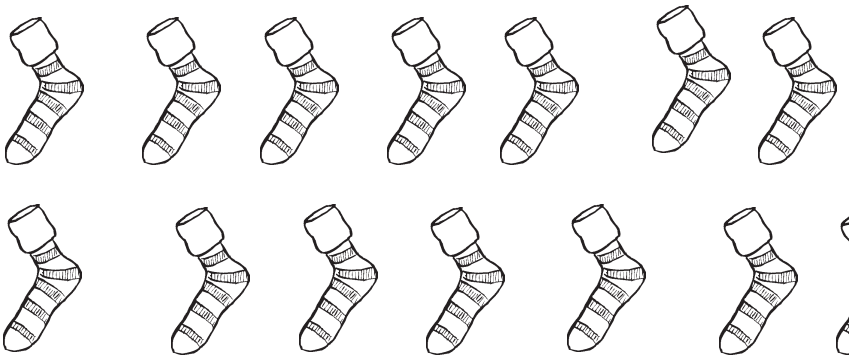
4 is half of 8 and 8 is double 4.

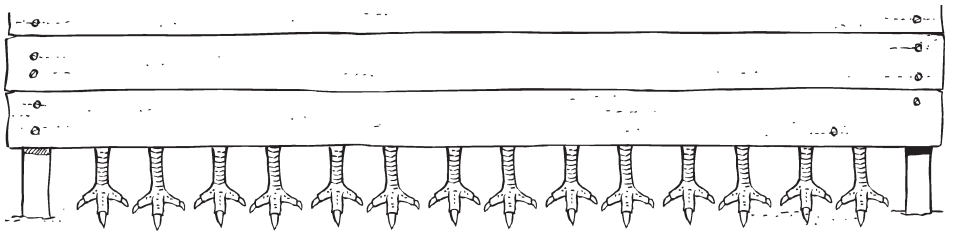
We are doubling and halving.

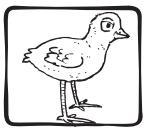
Workbook notes

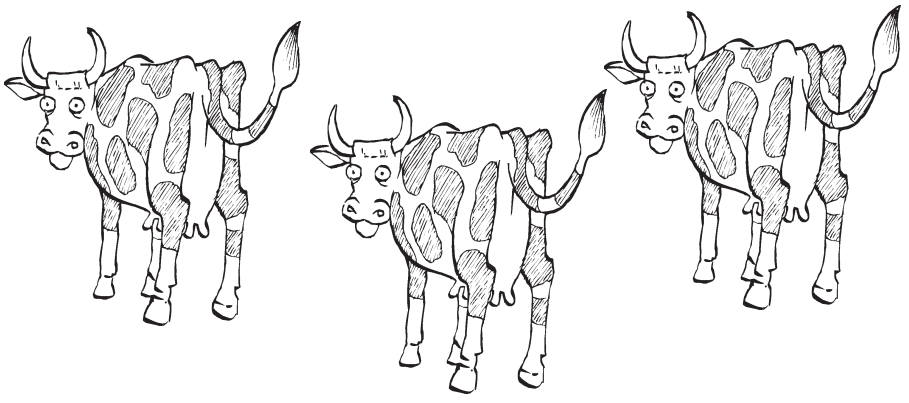


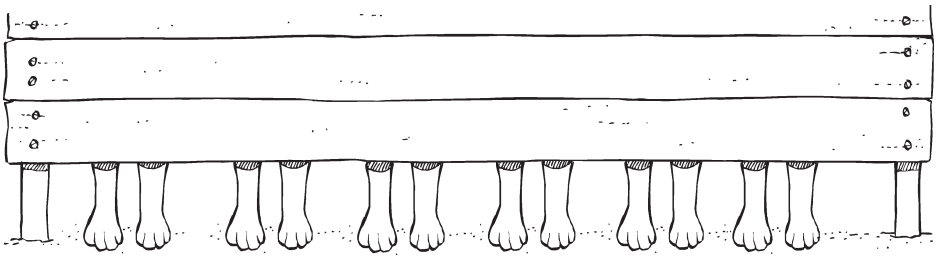
How many?

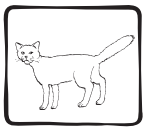




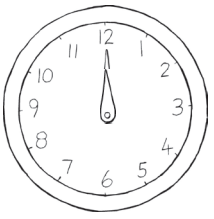




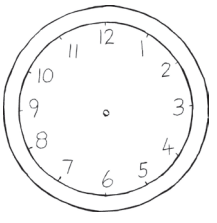




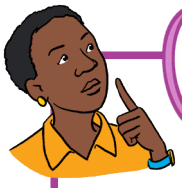
What is the time?



12 o'clock

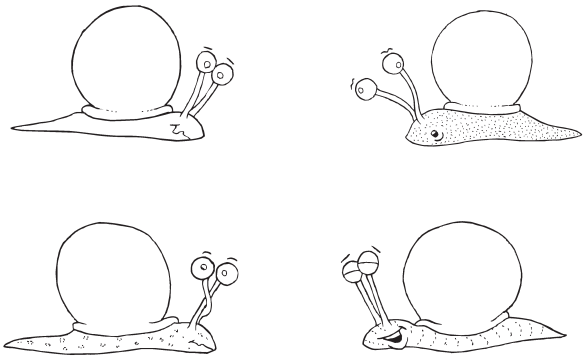


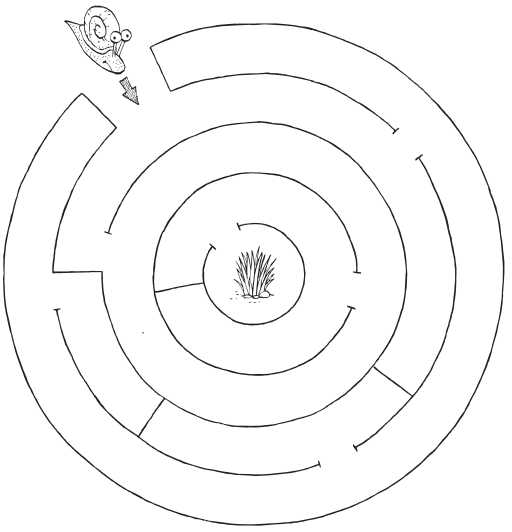
____ o'clock



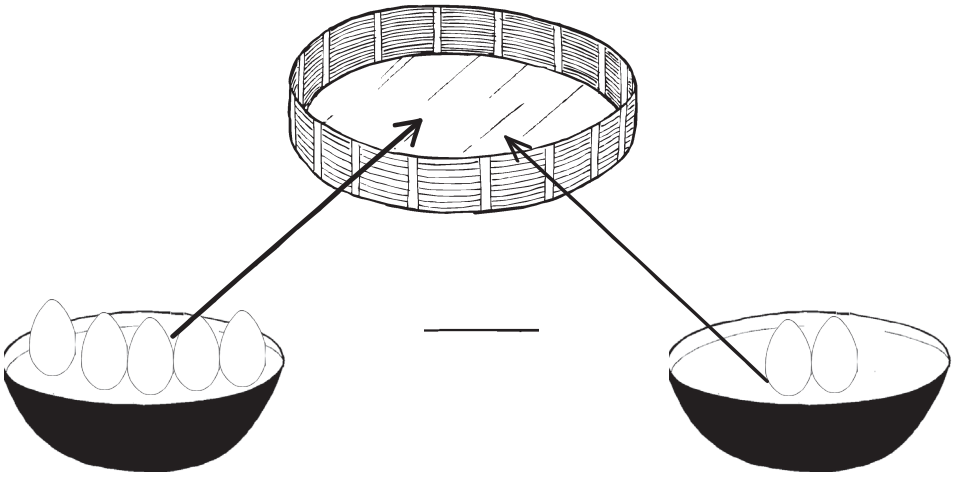
Refer to Workbook 0 p.13 to help you.

Help the snail to go home.

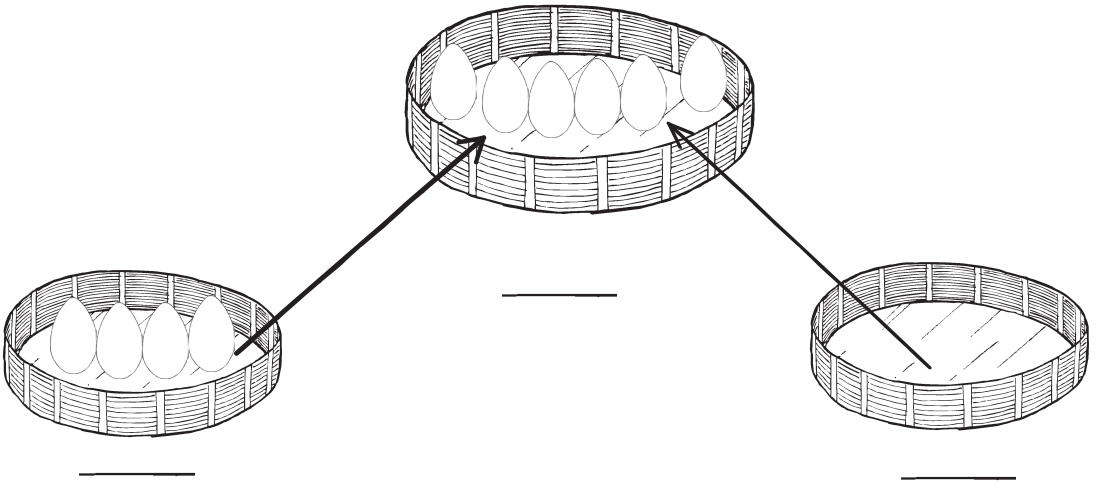




Complete the picture. Write the numbers.

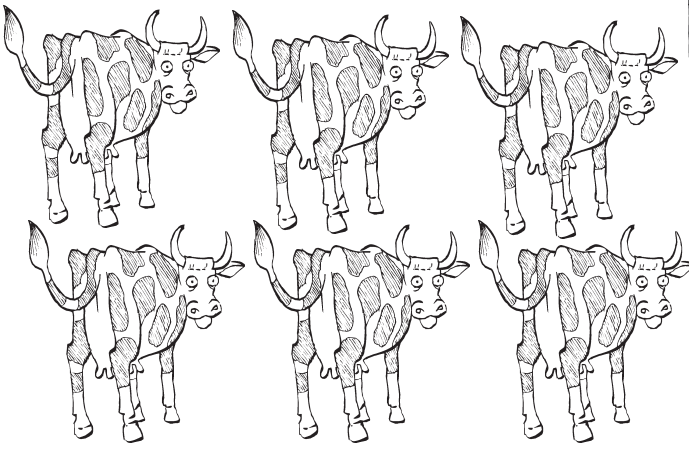


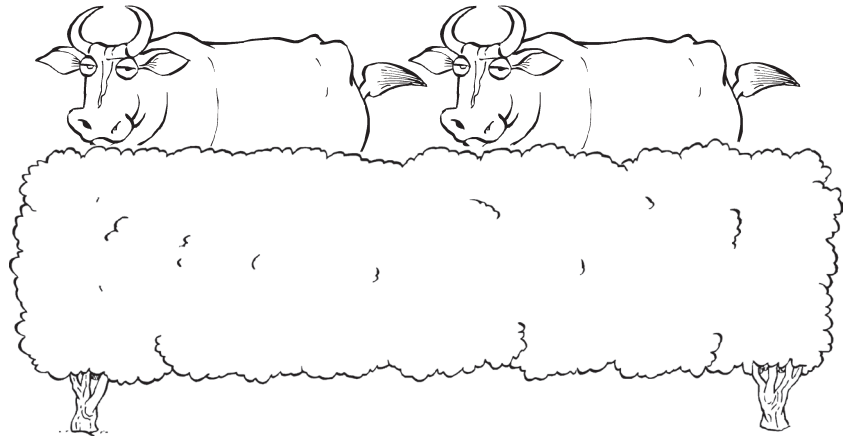
Complete the picture. Write the numbers.

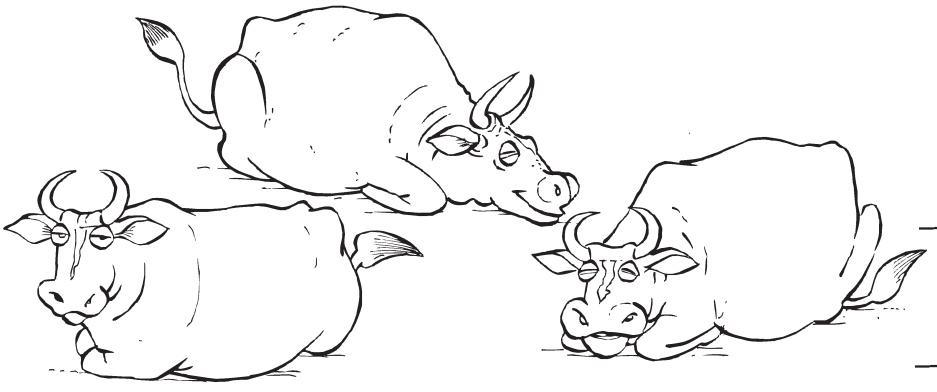


How many?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50







Write the numbers?

one	two	three	four	five

Things to think about

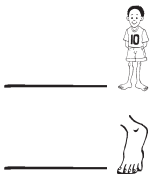
Workbook 0
Page 40

Hidden-part problems (leading to repeated addition)

First introduction to a related concept

Hidden-part problems are introduced in Workbook 00 p. 33 where children are required to work out how many children have 6 feet altogether (sharing).

How many?

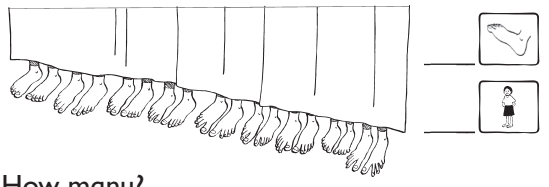


There are four more opportunities to practise this concept before the end of the workbook. New themes are introduced in the pictures which some children may find confusing.

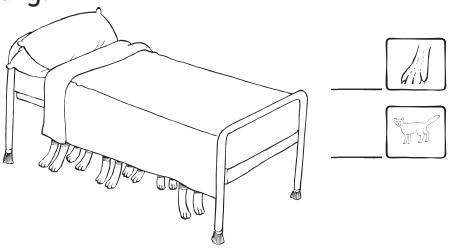
In the beginning of Workbook 0

On page 2 there is another opportunity to practise the hidden part (leading to sharing) concept.

How many?



How many?



Now:

Near the end of Workbook 0, on page 40, the concept is expanded to include the reverse situation where the number of groups is known (number of cows) and the number in each group is known (number of legs on each cow) and the total number of parts (legs) needs to be determined. A foundation for groups and repeated addition is being laid.

Introducing the concept

- *Revisit the first type of hidden-part problem where the total parts are known (number of feet), the number in each group is known (2 feet per child) and the number of groups (children) needs to be determined.*
- *Create a concrete situation in the classroom e.g. children behind a curtain with only their feet visible or use toy people and a piece of paper to cover everything except their feet.*

? *How many feet can you see?*
How many children can you see?
How many feet does a child have?
Can we use this information to work out how many children are behind the curtain/paper?
Let's see if we are right. (Pull back the curtain and count the children).

- *Do as many examples as needed (with guided questioning) until most of the children grasp the reasoning.*

Workbook notes

Now introduce the problem from a different angle. Use concrete props again but this time, cover the children's feet and legs. Follow a similar line ofquestioning:

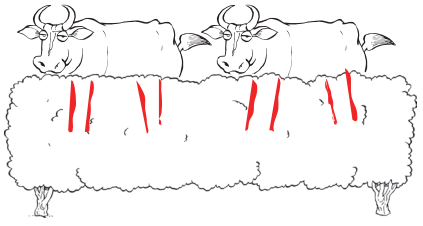
? How many children can you see? Can we see their feet?
How many feet does a child have?
Can we use this information to work out how many feet are behind the curtain altogether? Let's see if we are right.
(Lift up the curtain and count the feet).

<u>Check for understanding</u>
(parallel problem)
– as a class or group activity
Use numbers 6 and below initially.

Once you are happy the children understand the concept at a concrete level, <u>try a picture problem</u>. Draw a picture on the board and follow the same line of questioning. This time there is no opportunity to reveal the missing part so you will need to guide them in making a plan to determine the number of feet and to check if they are correct.

An example of a plan to work out the number of feet at a 2D level: drawing the missing parts onto the picture.

Now try a problem with animals that have 4 legs and check that children can generalise the concept of drawing the unseen parts.

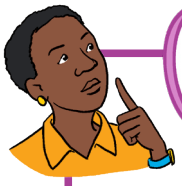


Once you are happy the children understand the concept at a pictorial level, allow them to <u>try the problems in the workbook</u> (p. 40)

<u>More practice:</u>
If you wish, keep practising this concept as a whole class or in small groups. There are no further workbook activities to try.

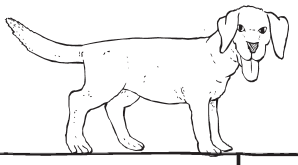
This process of introducing the concept and moving from concrete to pictorial and then being ready to try it in their workbooks might need several days for some children. Practising the process of problem solving is more important than grasping the concept – it is not an essential concept for Grade R.

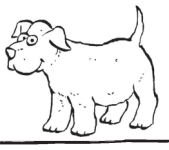
<u>Next stage in the development of the concept:</u>
In Workbook 1 and 2 (Grade 1), the children will have many opportunities to practise counting in groups with pictures and many opportunities solving problems on the mat. They should therefore eventually develop the understanding needed to work out an unknown quantity.

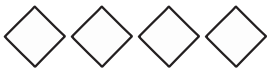
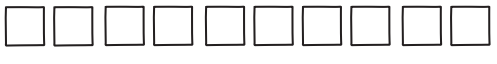


Refer to Workbook 0 p.33 to help you.

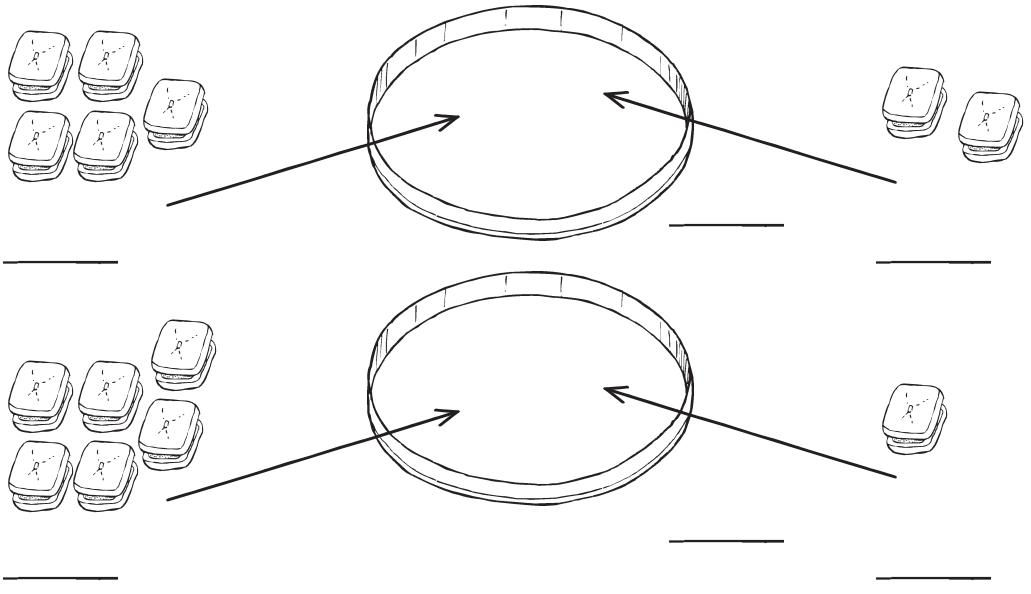
Halve.





Put the biscuits together. Write the numbers.



Write the numbers.

six	seven	eight	nine	ten

PATTERN

Overview

Studying patterns is at the heart of doing mathematics; in fact, mathematics can be described as the study of patterns.

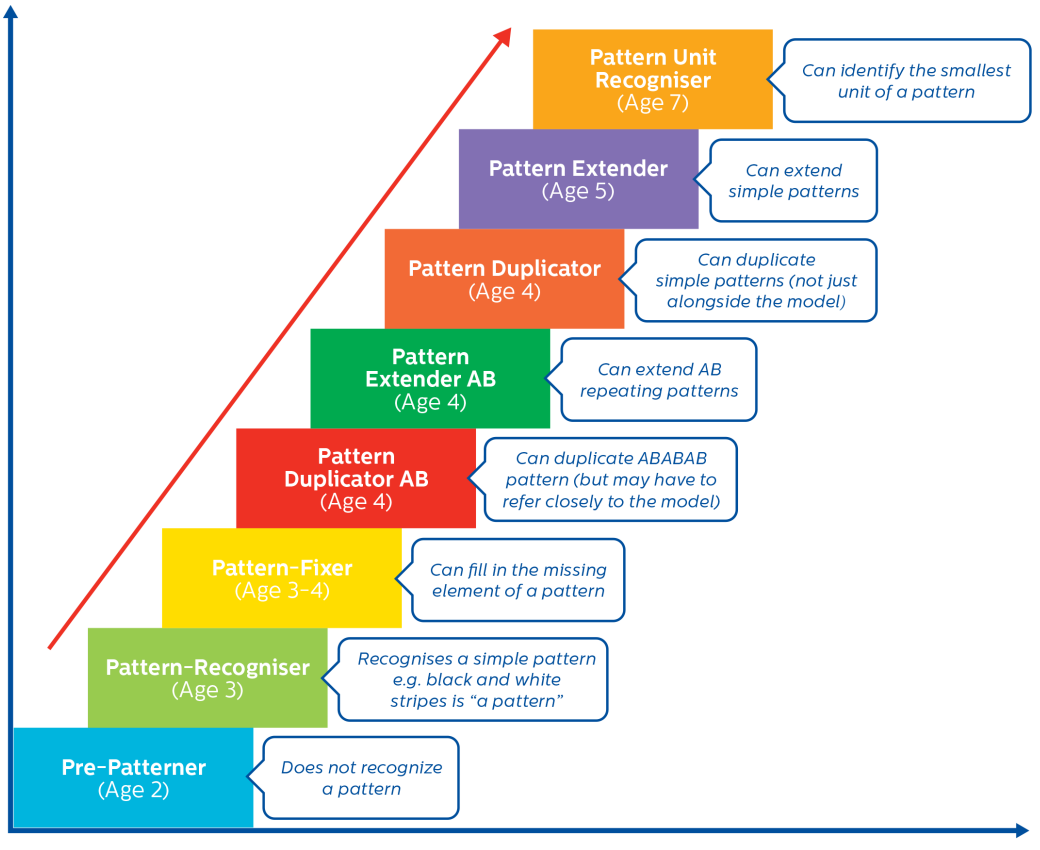
Patterns are relationships with some form of regularity or rule.

There is evidence that children's ability to recognise, describe and extend patterns is predictive of their future success in mathematics. If mathematics is indeed the study of patterns, then this should not be a surprise. However, the ability of children to recognise, describe and extend patterns can vary a great deal. For this reason, the study of patterns and patterning is a critical component of mathematics in the early years.

The challenge of school mathematics is not so much to 'teach patterns and patterning' as a topic, but rather, to make children aware of the regularity that is so much a part of their lives, to reflect on it and to develop the vocabulary with which to describe it.

Conceptual development

In the early primary years, we expect children to develop the ability to copy, describe and extend patterns in shapes, colours, numbers and other mathematical objects.



Developmental levels for patterning (adapted from Clements, Sarama & DiBiase, 2004)

The diagram depicts the developmental trajectory for the development of pattern that may be observed in Grade R children. The role of the teacher is to identify at which level a child might be at and to facilitate their progression through the levels to catch up with their chronological age.

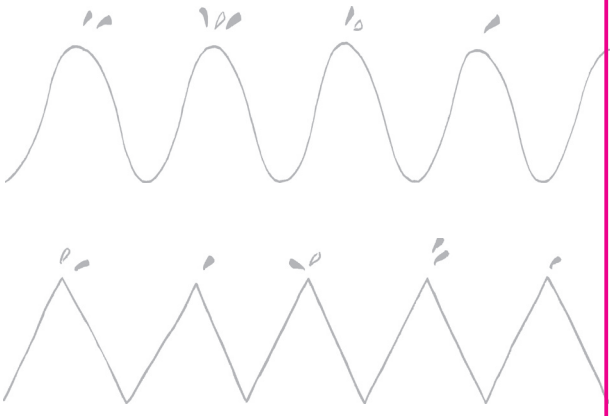
Pattern activities in Grade R

- There are several pattern activities in Workbook 00 and a few in Workbook 0.

  The aim of these pattern activities is to:
  - facilitate a motor experience of pattern and to improve pencil control.
  - create an awareness of pattern at a 2D pictorial level.
  - allow opportunities to practise copying patterns, extending AB and AAB repeating patterns and create their own patterns.

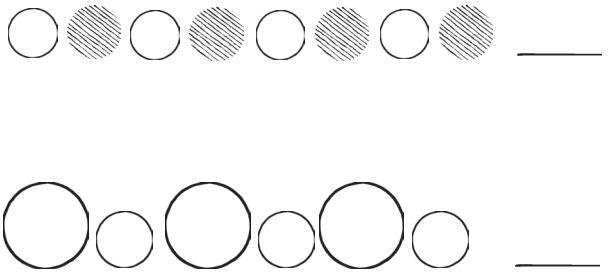
Workbook 00 p. 1

Trace the waves



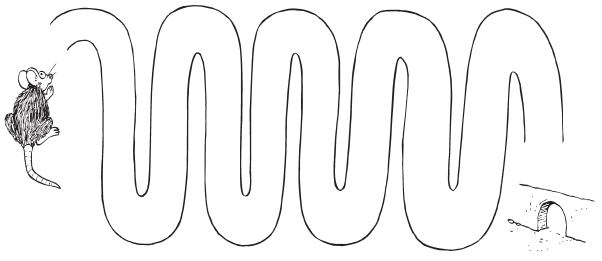
Workbook 00 p. 16

What comes next?



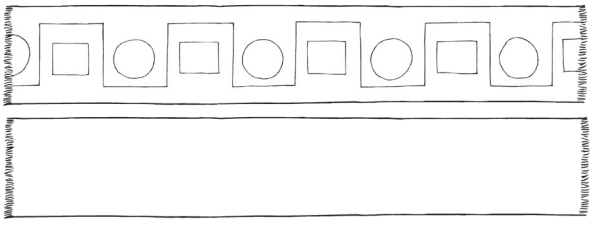
Workbook 0 p. 16

Trace the mouse's path.



Workbook 0 p. 39

Make them look the same.



Make your own.

- It is expected that most patterning activities will be hands-on using manipulatives such as beads, blocks, pom-poms and coloured sticks.
- Art activities also present opportunities for copying, extending and creating patterns.
- Grade R teachers can help children develop language and reasoning skills by encouraging them to describe the patterns they are making.

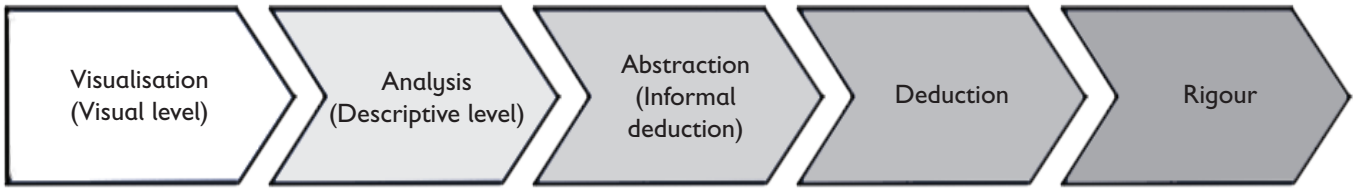
SPACE AND SHAPE

'Geometry begins with play.' - van Hiele, 1999

Conceptual development

The activities discussed below are informed by the research of Pierre van Hiele. According to van Hiele, there are five levels of geometric thought that are sequential and hierarchical. These include visualisation, description, abstraction, deduction and rigour (although it is unlikely that children in the early grades will move beyond the descriptive level). For children to function at any given level, they must have developed confidence at the preceding level. Progression from one level to another is largely based on instruction and experience rather than age or physical development.

Van Hiele's levels of geometric thought



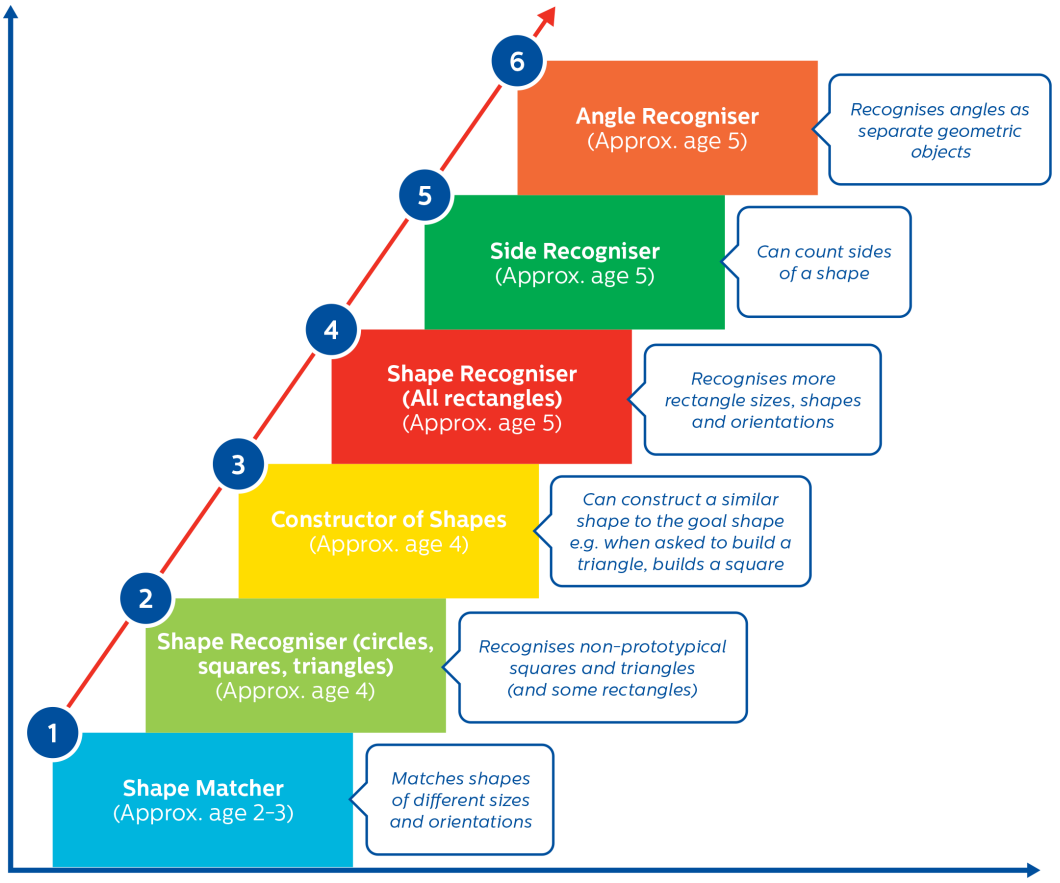
Visualisation (Visual level)	Analysis (Descriptive level)	Abstraction (Informal deduction)	Deduction	Rigour
Recognise figures by appearance alone. 'It is a square, because I can see it, it is a square.'	See figures as collections of properties but don't see the relationship between these properties.	Can order properties logically; they are deduced from one another.	Can construct proofs, understand the role of axioms and definitions, and knows the meaning of necessary and sufficient conditions.	Can construct proofs, understand the role of axioms and definitions, and knows the meaning of necessary and sufficient conditions.

For a child at the visual level of geometric thinking, figures and shapes are identified in terms of what they 'look like'. If asked why a square is a square, a child will say that it is a square because it looks like one. However, if the square is tilted so that its sides appear to be at a 45° angle, then the child may not recognise the shape as a square, instead, they may call it a diamond.

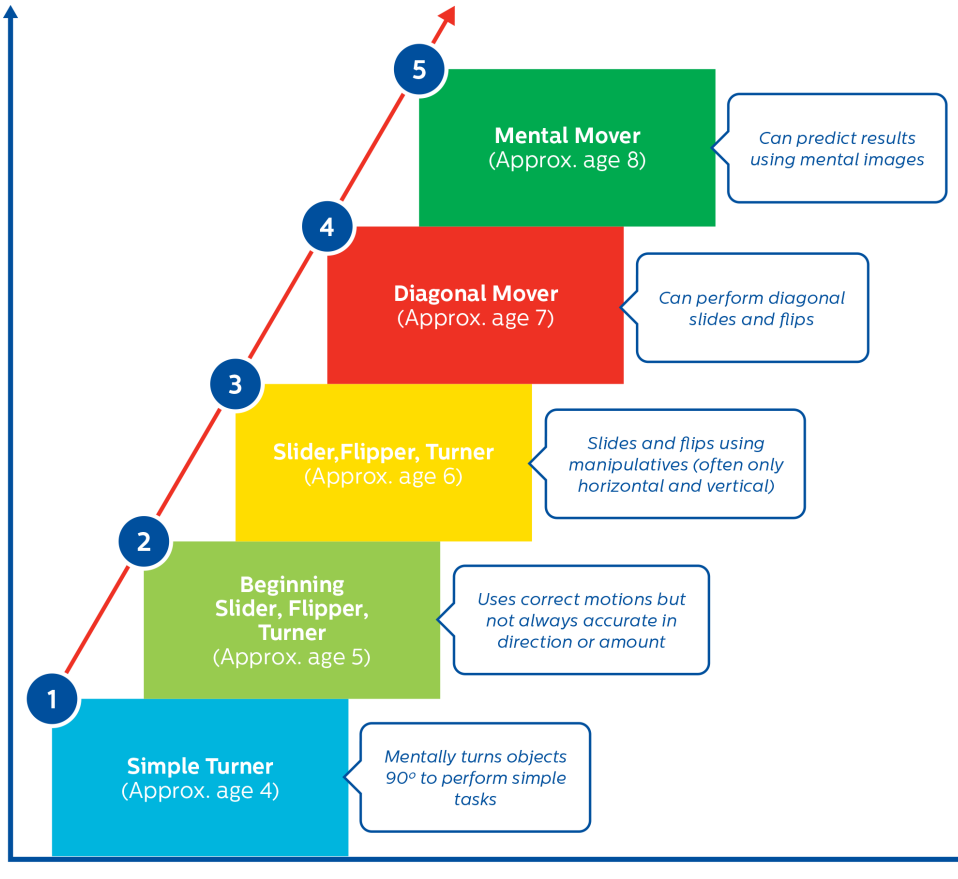
Teachers in the early grades typically work with children who are at the visual level of geometric thinking. The teacher's role is to create learning situations that develop the children's confidence in moving from the visual to the descriptive level of geometric thinking. Such learning situations can even lead to the informal deductive level.

The development of space and shape in Grade R

Consider the following extracts from learning trajectories for different aspects of Space and Shape that relate to levels of development that might be observed in the Grade R classroom. The aim is to accommodate for multiple levels of functioning, stimulating the children at the level they are at and gradually moving them through the levels in order to be age-appropriate by the end of Grade R.

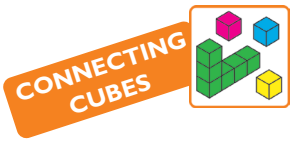


Developmental levels for recognising shapes (adapted from Clements, Sarama & DiBiase, 2004)



Developmental levels for spatial sense and motions (adapted from Clements, Sarama & DiBiase, 2004)

Space and Shape activities



Developing geometric thinking in the early grades is reliant on play with resources.

Many Grade R classrooms use activity stations and it is envisioned that a Space and Shape station could be one of the daily stations.
The NumberSense Activity Cards and resources have been developed to support the progression previously described.

The table below indicates a suggestion of the range of Activity Cards per resource that may be suitable for Grade R.

Grade	Mosaic puzzle	Tangram puzzle	Connecting cubes	Attribute blocks
Grade R	Guided play + Cards 1-4	Guided play + Cards 1-4	Guided play + Cards 1-2	Guided play + Cards 1-14

The NumberSense Grade R Activity Kit is under development and will contain a full set of Grade R-appropriate activity cards that utilise the existing manipulatives.

The role of the teacher in these activities might include:

- careful planning of tasks according to learning trajectories which may include less (closer to free-play) or more structure, such as a challenge or task cards.
- asking questions to direct the children's attention to the geometric qualities of a shape or to provoke certain types of thinking.
- introducing the children to the correct terminology and expanding their expressive language.
- engaging children in discussions that encourage problem solving and the explanation of their thinking.
- allowing opportunities for reflection and mediating these thoughts to advance their thinking.

Additional resources for hands-on Space and Shape activities

It is expected that Grade R teachers will already have a collection of manipulatives and learning materials in their classrooms and there is no need for NumberSense to duplicate these materials.

These materials included, but are not limited to:

- blocks and construction toys
- jigsaw puzzles
- ice-cream sticks and/or match sticks
- pegs and peg boards
- beads
- playdough
- 3D recyclable objects
- sand and water play equipment
- art resources (2D and 3D)

All of these materials are available locally from multiple sources. Activity cards can be sourced online, downloaded and printed to give direction to children's play and to support their progression to higher levels of geometric thinking.

Capacity, Length, Weight, Area

The development of an understanding of measurement follows a similar progression irrespective of the attribute (capacity, length, weight or area) being measured.

1. First, children must develop an awareness of the attribute. This is achieved through comparing, ordering and matching different objects with the attribute.
2. Next, children must learn to quantify the attribute accurately, using units which progress from non-standard and informal units, to more formal units.
3. Finally, the children must use standard units and perform calculations in measurement contexts.

Developing an awareness of attributes

To support children in developing an awareness of the different measurable and comparable attributes that an object has, early activities focus on children comparing objects by filling them in order to develop an awareness of capacity, using a balance to compare objects in terms of their weight, and comparing the lengths of objects by both direct comparison and using informal length units.

Quantifying attributes

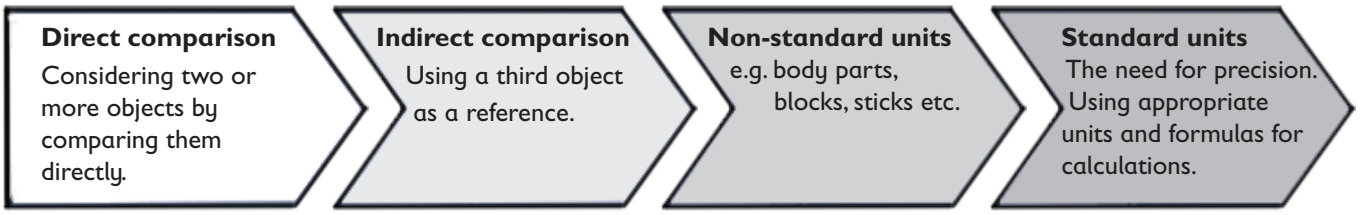
Objects can often be compared in terms of one or other attribute by means of direct comparison. This can be done by holding them next to each other or picking them up to feel which is heavier and which is lighter. As the difference in the attribute being compared becomes smaller, so it becomes harder to compare the objects by direct comparison and it helps to be able to quantify the attribute. Initially, this is done using informal units. Quantifying the attribute also helps to answer questions such as, 'How much more does the larger object hold?', 'How much heavier is the larger object?' or, 'How much longer is the larger object?'.

The workbook activities (in later grades) are carefully structured to help children develop an awareness of how the choice of measuring units can impact on both the efficiency (ease) with which the attribute is measured and the precision of the measure. Throughout the activities, children should be encouraged to estimate before measuring. Initially, these estimates may be no more than guesses. Estimating helps the children develop a sense of the 'muchness' of the unit. Children should also reflect on the appropriateness of the units used to measure an attribute. For example, using a cup to determine the volume of a bucket is probably more sensible than using a tablespoon. The activities also support the development of the awareness that using non-standard units (e.g. hand spans, arm lengths and paces for length) creates difficulties and that there is value in using standardised units.

Using standard units and calculations

The value of using standardised units often follows from the experiences of different people using different informal and non-standard units to measure and compare the attributes of objects. However, children should also realise that the choice of standard unit is for the most part arbitrary and the metric system as we know it is no more than a widely-accepted convention to use specific units to measure certain attributes.

The activities in the workbooks (in later grades) provide the practical experience and opportunity for children to develop an understanding of measurement in a meaningful way. The progression of the activities has been deliberately planned so that, with the appropriate facilitation and guidance of the teacher, the activities reveal the mathematics in a meaningful way.

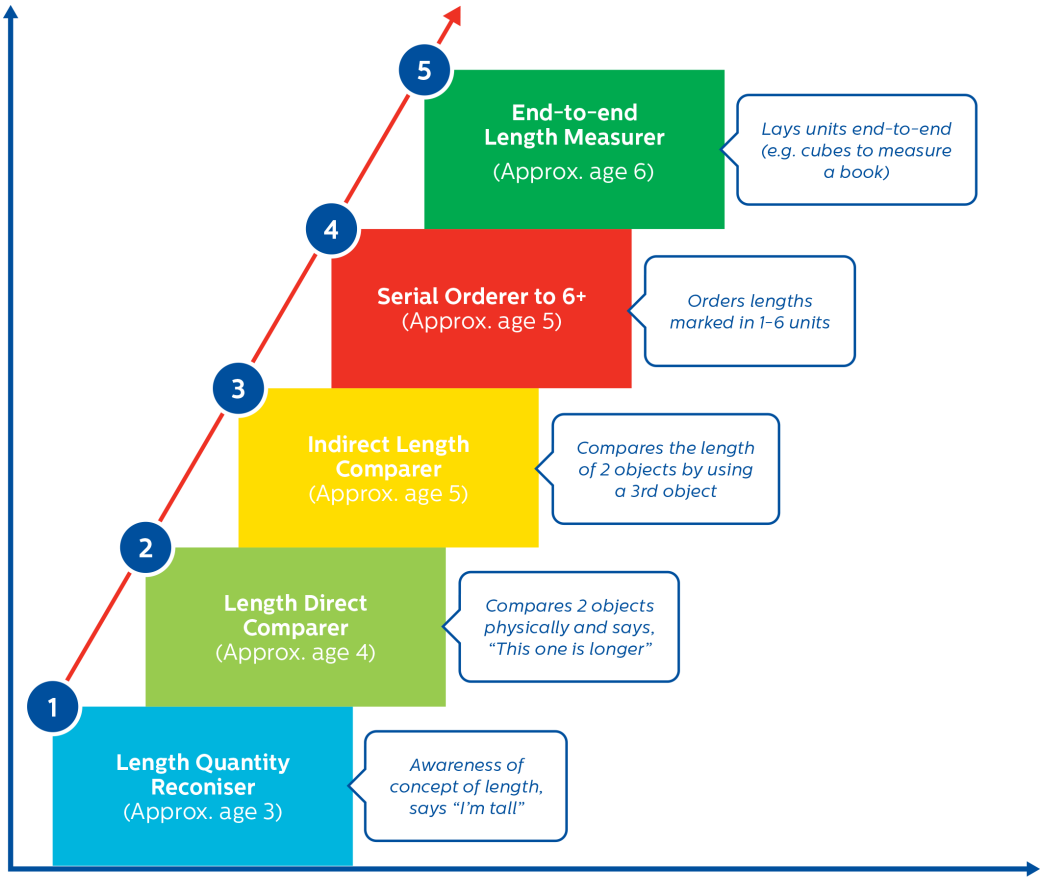


Measurement activities

The focus of the activities in the later workbooks is on the children gaining experience and confidence in filling, weighing and measuring a variety of containers or objects, using a variety of units. Through these practical activities, the children will experience measuring larger containers with smaller units, as well as measuring smaller containers with larger units.

Children should develop an awareness of the different measurable and comparable attributes (capacity, weight and length) that an object can have. They should also develop an awareness of the suitability of the measuring unit. Larger units are quicker but less accurate and not as suitable for smaller containers. However, smaller units are more time-consuming - especially as the size of the container increases - but more accurate.

Measurement in Grade R



Developmental levels for measuring (Adapted from Clements, Sarama & DiBiase, 2004)

**Sample trajectory for length*

In making comparisons, the children should develop the awareness of the attribute that is being measured. They should also begin to notice that the tallest container may not necessarily hold or weigh the most and that the heaviest container may not be the tallest or hold the most etc.

The trajectory for length on page 28 provides a helpful developmental trajectory for measuring. Grade R children may be functioning at any of these five levels. The role of the teacher is to identify at which level a child might be at and to facilitate their progression through the levels to catch up with their chronological age.

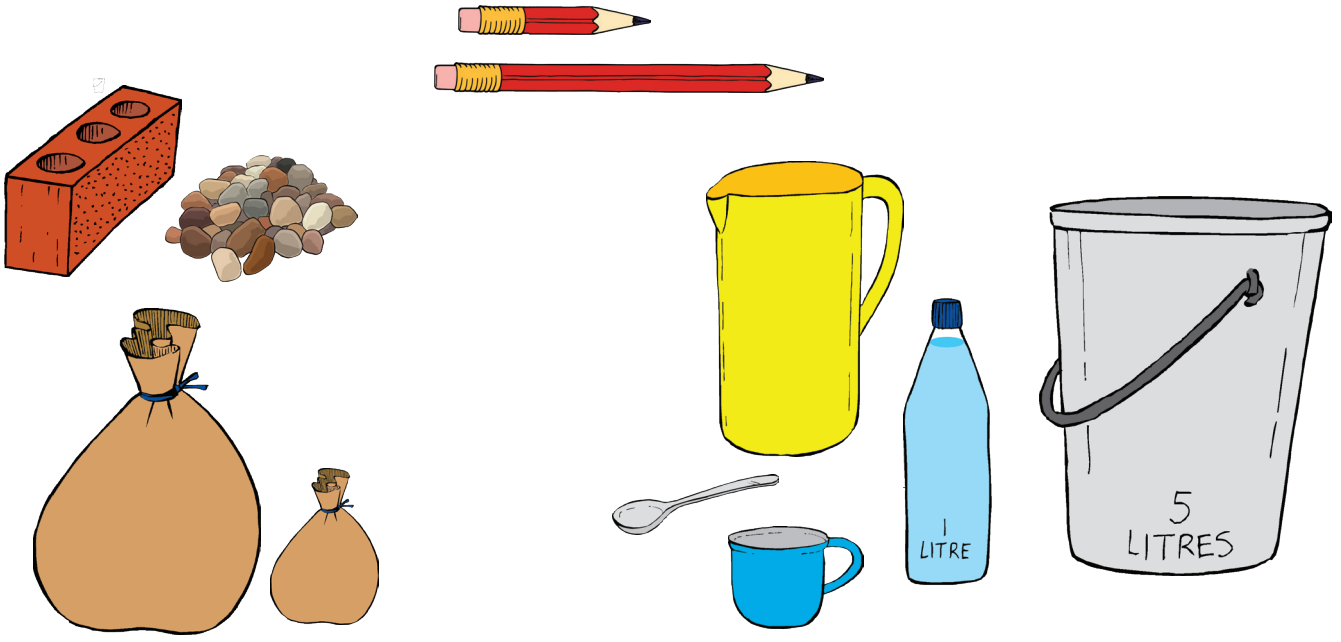
In Grade R, the goal of measuring is to answer the question, 'Which is larger/smaller?' We do this by moving from activities where two objects can be directly compared to those that need to be indirectly compared. The measuring skills being developed include comparing, ordering and matching according to attributes.

Direct comparison involves considering two or more objects and comparing them in terms of the relevant attribute.

Indirect comparison involves using a third object as a reference to compare two other objects that cannot be directly-aligned or compared. Because it is not always possible to compare objects directly, we use a different (but common) object that can be compared directly to each of the objects to act as a reference.

Resources

It is useful to have a range of boxes, bottles, containers and objects that can be used as informal units. For every stage of development, the container selection should include containers that are very different in length/capacity/weight so that children are able to estimate which is 'bigger/smaller'. By simply looking at the containers, the children should be able to order and compare. A homemade balance scale is also useful (see image). If you do not have a balance scale, it is easy to make.



Grade R measurement activities

- There are no measuring activities in Workbook 00 and 0. It is expected that measurement concepts will be developed through concrete, play-like activities and through the daily activities of the Grade R classroom.
- The Grade R child needs to learn the language of measurement which can be supported through the teacher's questions.
- Sand and water play are very important in helping children develop a sensory-motor concept of capacity, weight and volume. Carefully-considered resources provided to Grade R children can assist in encouraging different types of skills development.
- Block play helps to develop a child's sense of length and height.
- For sand, water, and block play, the teacher can consider setting challenges for the children. For example, 'Which of these containers holds the most/least water?' or 'What is the tallest tower you can make using these blocks?'

Time

The main focus in the early years is to develop children's understanding of how the world is organised through time. This happens by providing children with a concrete experience of time on an everyday basis. Children in Grade R and at the beginning of Grade 1 who have not yet been exposed to the structured school environment, would mostly have developed an understanding of social time at home – 'I go to bed when mom says so', or 'When I finish my drawing, I will have juice'. According to Piaget (1964), children at this age are in the pre-operational stage of development and cannot engage in logical thought, separate ideas, make mental representations of ideas or reverse the sequences of events. This limits their understanding of time.

As children start attending school (5-7 years old), learning about time should be aimed at developing formalised ideas about time and the understanding that, culturally and socially, life is structured through time. The focus is on learning the language of time and talking about time in the context of the routines at school. Learning time words such as before, after, morning, afternoon, evening, tomorrow, yesterday, last week, early, late, longer and shorter, is of particular importance. Sequencing events is another important part of learning about time that is supported by the learning of the language of time.

At this stage, one can start to associate schedules with clock time: 'Break is at 10 o' clock', 'School starts at 8 o' clock', etc. Children at this age will have difficulty understanding elapsed time and time in the future. Awareness of when events occur is still limited. For instance, a child might not know today is their birthday unless someone tells them.

Children in Grade 2 and 3 (age 8 -9) are beginning to move towards the concrete operational stage. They have a better understanding of so-called physical time that is measured with clocks and watches. They are also ready to start keeping track of time using clocks and watches and to learn about units of time.
At first, telling the time using an analogue clock is very confusing for children. The numerals on an analogue clock have dual meanings. Not only do they refer to the hour but also to multiples of five minutes. Having a clock on the classroom wall that the teacher points to regularly during the day and says 'It is now eight o' clock' etc. plays an important role in the development of an understanding of time.
Although children are able to think more logically at this point, working with the time concept should still be concrete and related to their day-to-day experience of time. Children can be given hands-on-activities such as using stop watches to time the baking of a cake, how long it takes to eat lunch, etc. This develops the time concept and its measurement in a concrete way.

Time activities

It is with this background in mind, that we have developed a set of cards that can be used to facilitate the learning of time in the Foundation Phase. Teaching with the cards as a resource takes the approach that the learning of time is experiential, with a strong focus on developing children's ability to sequence events. It also focuses on learning the language of time and aims to develop children's understanding of time as a real-world experience.

The NumberSense Event Cards are designed to offer tasks and games that develop children's ability to sequence events, whereas the NumberSense Telling Time Cards are aimed at developing children's knowledge of physical time.

The Time card tasks are intended for small group teaching on the mat and to be used in conjunction with the workbook pages. The complexity of the tasks and games increases as more cards are added.

The NumberSense Event Cards illustrate events that generally form part of a child's daily routine. This provides familiarity with the duration of events and for an everyday point of reference. The activities start by sequencing a small number of events and, as more events are added, the activity becomes more interesting and complex. The tasks centre on sequencing, duration of events and the sorting of events according to the time of day. The cards each depict an event that represents increasingly complex situations and come in four sets arranged by colour - red, blue, green and purple. In total there are 28 event cards that can be used in the tasks and games suggested in the workbooks.
In Grade R, the concept of duration is expressed as comparison (and not a measurement of time). For example, sleeping takes longer than bathing.

The NumberSense Telling Time Cards add the dimension of physical time. The purpose of this set of cards is for children to learn to tell time using analogue and digital clocks in 12-hour time and to develop children's ability to associate hours of the day with events. Children progress from telling the time in hours, then half hours, quarter hours and finally, five-minute intervals. They become familiar with the different time formats as they match the analogue clocks, digital clocks and time in word cards. They practise elapsed time by comparing the time on clock faces. These also come in four sets that represent increasingly complex measures of time. The sets are arranged by colour and each set of cards consists of a card with an analogue clock, a card with a matching digital clock face and one with the matching time given in words for each of the times in the set. There is a total of 52 cards in a pack.

In Grade R, the focus is on exposing children to whole hours e.g. 8 o'clock. Only the red Telling Time cardss, which depict whole hours for analogue and digital, are used. There is no expectation of being able to tell the time (even in whole hours) or understanding the parts of hours. The main aim is to develop an understanding of real-world time of day and attach a basic understanding of whole hours. For example, a Grade R child might have a partial understanding that they wake up at 6 o'clock in the morning; school starts at 8 o'clock; break time is at 10 o'clock; school finishes at 1 o'clock; supper is at 6 o'clock and bedtime is 8 o'clock in the evening.

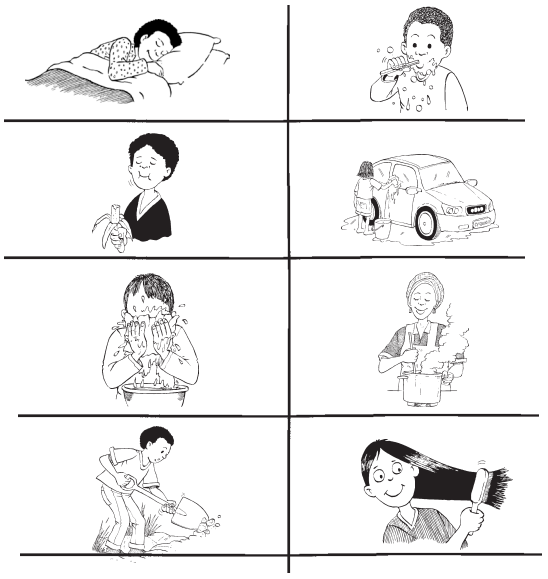
In later grades, the activities throughout the workbooks are repeated frequently, changing the cards and adding more cards as the children gain confidence with the different concepts associated with time.

Grade R time activities

- Time should be referenced throughout the day and not restricted to mathematics lessons. For example, 'In 5 minutes it will be 10 o'clock and that is break time. Let's pack away and get ready for break.' Or, 'We've done our good morning song, what do we do next every morning?'
- There are a few workbook activities. In Workbook 00, on page 31, children are asked to compare the duration of certain activities and colour the one that takes the longest. Workbook 0 p. 7 introduces the o'clock terminology and then there are 6 opportunities to practise (p. 12, 17, 19, 25, 27, 34).

Workbook 00 p. 31

Which takes the longest?
Colour in the picture.



Workbook 0 p. 7

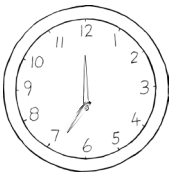
What is the time?

1 o'clock

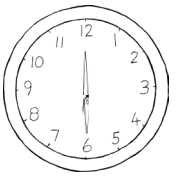
2 o'clock

Workbook 0 p. 19

What is the time?

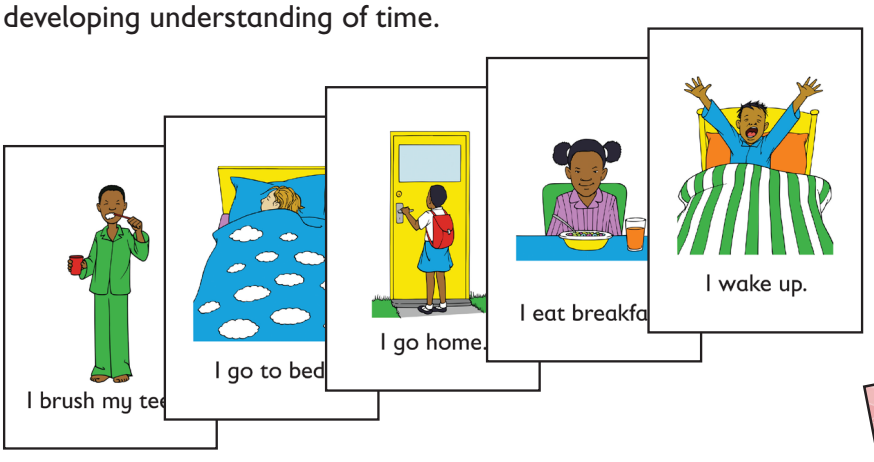


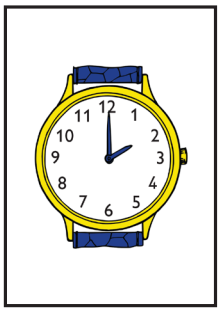
_____ o'clock

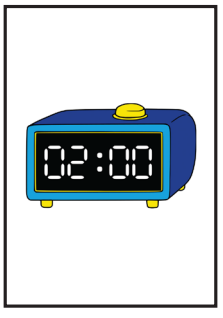


_____ o'clock

- The red NumberSense Telling Time Cards can be used to help children practise telling the time in whole hours.
- Teachers can promote conversations about the sequence of events and comparative length of activities using vocabulary such as 'before, after, later, then' and 'longer, shorter, quicker, slower, sooner, later'. The red NumberSense Event Cards can be used for these discussions. Children can sort the events into things they do in the morning, afternoon and evening. A few event cards can also be used for an ordering activity where children order them from the shortest to the longest in duration.
- Daily calendar discussions (years, seasons, months, weeks, days) help contribute to the Grade R child's developing understanding of time.







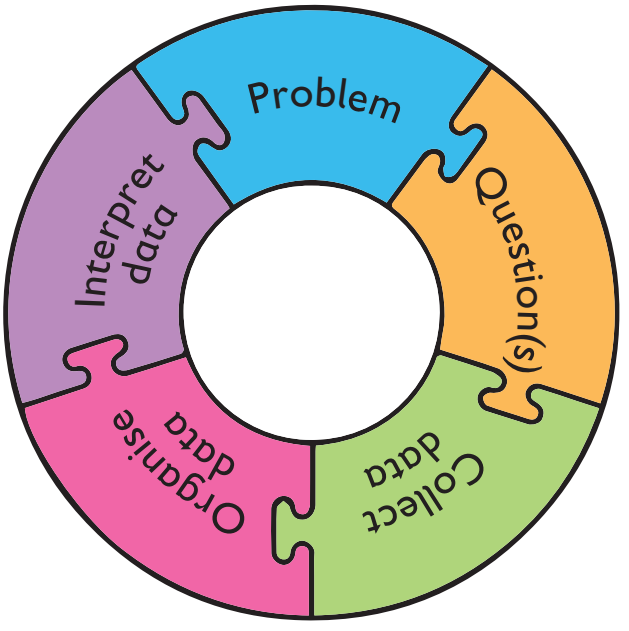
DATA HANDLING

The data handling component of a typical school curriculum draws attention to the different stages of a problem-solving approach in which data is collected to answer a question. From comparing the price of a particular commodity across a range of shops to determine the 'best deal', to canvasing the opinion of friends and family about a restaurant before deciding to go there, we collect data to make decisions every day.

As individuals, the decisions we make based on the data we collect may not be high stakes and may not have significant consequences. However, the stakes are very high and the consequences significant when, for example, drug companies collect data to make decisions about the efficacy of new drugs. The study of data handling in the mathematics curriculum should be focused on developing an awareness of the role that different choices made at each stage of the process have on the outcome of the process.

In the early years, it is enough for children to develop an understanding that data can help us to better understand situations and make informed decisions. As children move through school, they need to become increasingly aware of the choices that they make at each stage of the process and how these choices impact on the conclusions that can be made.

From the early to the later years, we want children to experience data handling as a process or sequence of events - a cycle. Classroom activities should expose children to all of the elements in the cycle in order for data handling to be meaningful. In particular, when using data to solve a problem, they should be exposed to each of the events of the data-handling cycle together, not separately. In other words, we don't do one activity where we collect data and another separate activity where we draw graphs. Instead, we structure learning activities to include all of the stages in the cycle: collecting data, organising (summarising and representing) data, and interpreting the summarised data - all with the purpose of solving a problem or answering a question.



Usually, the data handling cycle starts with a problem - something that needs to be solved or better understood. A question that needs to be answered in order to solve the problem is then identified. Next, appropriate data is collected in order to answer the question. Once the data is collected, it may need to be organised in a way that makes it easier to answer the question. This usually involves summarising to reduce the amount of data and representing it in a way that makes the trends clear to see. Finally, the data is interpreted in terms of the problem and question that started the process so that the question is answered and the problem solved.

Data handling activities

During activities, teachers need to continuously ask children questions about their data. For example, 'How has the data been sorted/organised?', 'Why has it been represented in the way that it has?' etc. In answering questions based on the data, children need to develop the skill of making statements that they can then justify in terms of the data.

When children start sorting according to their own criteria, they might sort objects:

- 'just because' they belong together. At this stage, children are typically at the visualisation stage of geometric thinking (van Hiele, 1999).
- according to certain attributes such as shape, colour, size or the material from which the objects are made, etc.
- according to function: objects that we eat, wear, play with, throw away etc.

It is very important that teachers support children in explaining the reasons they used to sort the objects in the way that they did. Explaining their decision forces children to reflect, and it is through reflection that they develop their understanding.

Children will typically begin sorting by one characteristic at a time. This should be encouraged as a regular activity with a variety of materials with which to sort. Progression is also important in moving from sorting according to a single attribute and later introducing more complex examples whereby children then sort according to two or three attributes.

Grade R activities

- There are a few Data Handling activities in Workbook 00.

Workbook 00 p. 2

Colour in the big shells.

Workbook 00 p. 15

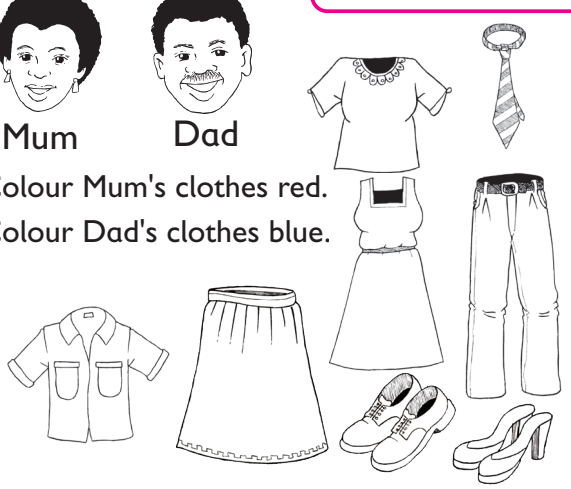
Find the other shoe.

Workbook 00 p. 13


Mum


Dad

Colour Mum's clothes red.
Colour Dad's clothes blue.

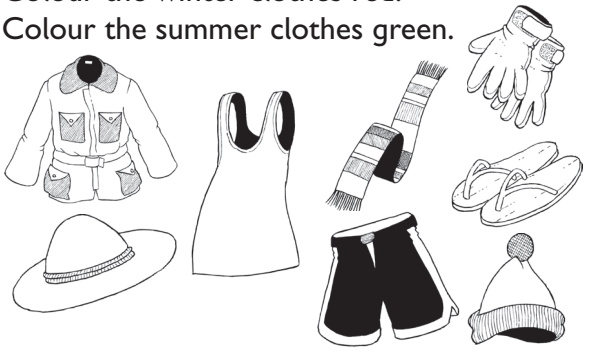


Workbook 00 p. 14

Winter
Cold

Summer
Hot

Colour the winter clothes red.
Colour the summer clothes green.



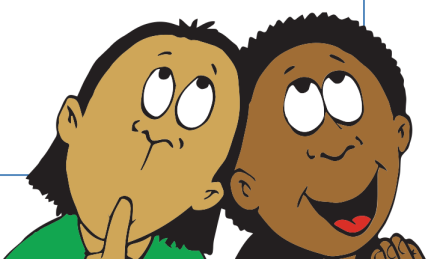
- In Grade R, most Data Handling activities will involve hands-on sorting of concrete objects or pictures. For example:
  - sorting manipulatives according to colour, shape, size.
  - sorting animal pictures into animals that live on land and animals that live in the water.
- Consider using sorting and classifying activities in Life Skills lessons.
- Encourage children to express the reasons for their thinking. For example, 'How did you sort these?' 'Why?' 'Is there another way to sort them?'
- The focus here is on the children gaining experience and developing the skill of interpreting data by engaging in discussion which is mediated by the teacher.

Notes

These planning resources have been designed for Grade 1 - 6

Understanding the page: checklist

	PROMPTS	MY NOTES
PRE-TEACHING QUESTIONS	**Things to think about:** • What learning am I supporting (task: the page)? • What learners am I supporting? • What resources do I need to facilitate learning? • What reflection or discussion would be useful? – For what purpose? • What responses/solutions do I anticipate?	
PRE-TEACHING QUESTIONS	**Differentiation:** Who and what will need scaffolding, consolidating, extending? • Does this mean: – Whole class? – Mixed groups? – Streamed groups? • Do I need to adapt the task? – Do I need a parallel/similar problem? • Do I need to use supporting resources? • Do I need to give more input prior to the activity?	
TEACHER-LED	**What is the purpose of the task?** • What mathematics does the page intend for children to notice? • Have we done something like this before? • Will this be problematic for some/all? • Can I make links to similar, previous tasks?	
TEACHER-LED	**Anticipated/desired strategies (age- and grade-level appropriate):** • What strategies do I anticipate that the children will use? • Which strategies do I aim to encourage?	
INDEPENDENT WORK	**Monitoring:** • Will I need to pair certain learners for this? • Will I need to monitor for understanding during the task? • Will I need to identify different strategies being used to draw on during the reflection session? • Which learners can I select to explain their work?	
REFLECTION	**Discussion (can include marking):** • Where/how can I make links to previous tasks? And how can I build on these? • Use the question bank to select specific questions to ask about the task/page (to reveal what we want the children to notice). • How and when will marking take place? • Which activities will corrections be useful for?	



Understanding the page: key prompts

As part of your preparation, complete the workbook page and these key prompts.

Notice

What mathematics does the page intend for children to notice?

Plan

Parallel activities: Which activities on the page require a similar activity to sufficiently prepare the children for the page by scaffolding, checking for understanding (consolidating) or extending?

Anticipated/desired strategies (age- and grade-level appropriate): To think about while planning: what strategies do you anticipate that the children will use?

Reflect

Use the question bank to select specific questions to ask about this page during the reflection session (to reveal what we want the children to notice).

Question Bank

Questions that scaffold thinking

1. What did you already know that helped you?
2. What did you notice?
3. Have we seen this before?
4. How does that help us with this activity?
5. How is this the same and how is it different?
6. What has this activity taught us?
7. If you don't know how to do this, what do you know that can help you?

Questions that build on learners' ideas develop understanding

8. What strategy did you use?
9. How did you decide what to do?
10. Can you explain how/why it worked?
11. Did anyone use a different strategy?
12. Why did you do it differently?
13. Which of these ways will always work?

Questions that develop reasoning

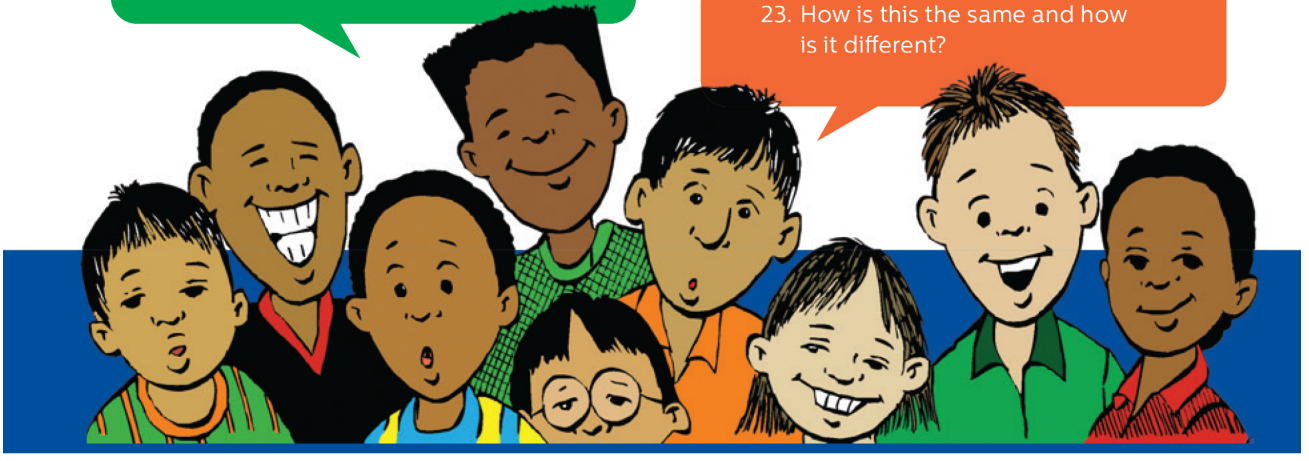
14. How did you do that?
15. Can you explain why it worked?
16. What did you notice?
17. If you do know this, what else do you know?

Questions that strengthen the concept (though the noticing of the connections between different mathematical representations)

18. Have we seen this before?
19. How does that help us with this activity?
20. How is this the same and how is it different?

Questions that reveal misconceptions/errors

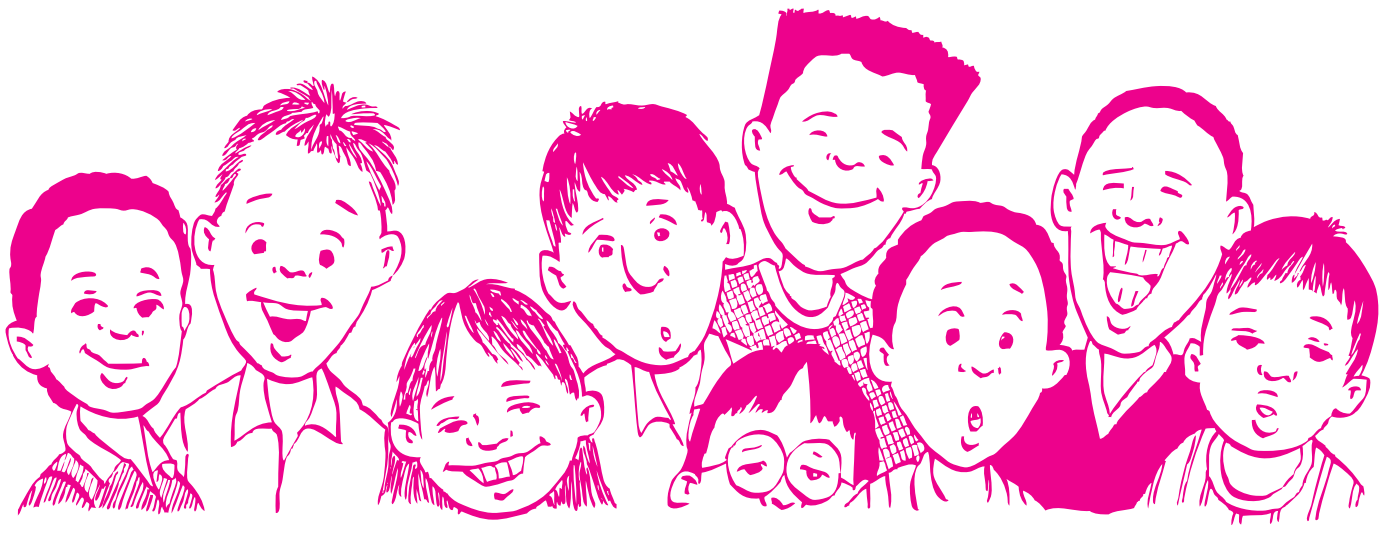
21. Have we seen this before?
22. How does that help us with this activity?
23. How is this the same and how is it different?



NUMBERSENSE MATHEMATICS PROGRAMME RESOURCES PER GRADE

Grade	NumberSense Workbooks	GeoGenius Construction Kit (super)	GeoGenius Visualisation Kit	Activity Kits and Resources	Additional Resources
R	Grade R: 00, 0			Grade R Activity Kit, which includes: - Number & Pattern resources - Space & Shape resources - Measurement (time) resources	Teacher Guide
1-3	Grade 1: 1, 2, 3, 4	1 per class		1 x Activity Kit (super) per class, which includes: - Activity Kit (standard) which contains all the Activity Cards - Attribute Blocks x 3 packs - Connecting Cubes x 3 packs - Tangram Puzzle x 1 pack (10 sets) - Mosaic Puzzle x 1 pack (10 sets)	Teacher Guide Mental Arithmetic booklets
	Grade 2: 5, 6, 7, 8				Teacher Guide Mental Arithmetic booklets Geoboards
	Grade 3: 9, 10, 11, 12		2 per class (optional)		Teacher Guide Mental Arithmetic booklets Geoboards

* Note: We do not stock geoboards, but these can be purchased via Takealot, Greenbean or Smile. We suggest the 5-by-5 square pin grid array.



Brombacher and Associates cc

www.brombacher.co.za
info@brombacher.co.za

Unit E23, Prime Park,
Mocke Rd, Diep River, 7800